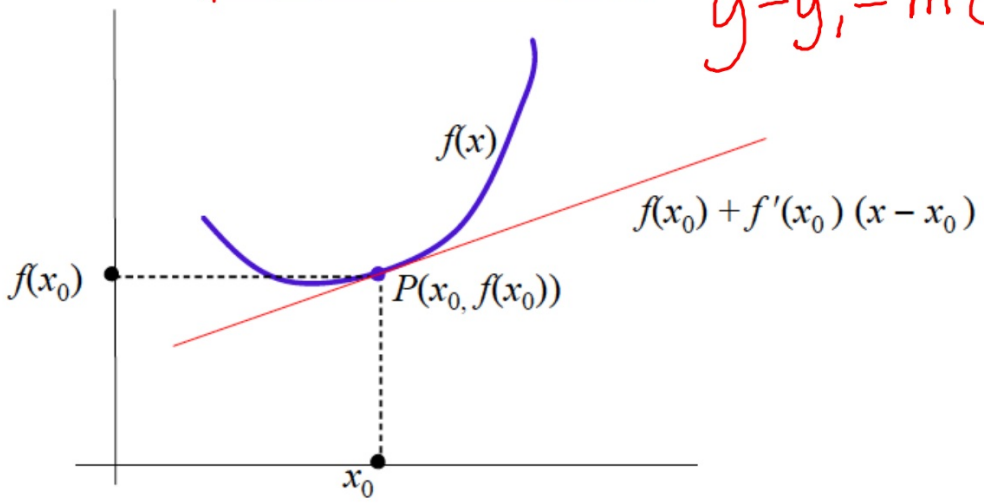
Local Linear Approximation
aka
Tangent Line Approximation

Local Linear Approximation

The equation of the tangent line to the graph of the function $f(x)$ at the point x_0 is $y - f(x_0) = f'(x_0)(x - x_0)$

$y - y_1 = m(x - x_1)$



Example. Use local linear approximations to approximate the quantity $\sqrt{80.9}$

Is this an over or under approximation?

$f(x) = \sqrt{x}$ $\quad (81, 9)$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad y - 9 = \frac{1}{18}(x - 81)$

$f'(81) = \frac{1}{18}$ $\quad y - 9 = \frac{1}{18}(80.9 - 81)$

$y - 9 = \frac{1}{18}\left(-\frac{1}{10}\right) = 9 - \frac{1}{180} = 8\frac{179}{180}$

over approx. $f''(81) < 0$ (concave down)

1. Write a tangent line for the nearby point.
2. plug in the 'x' value and find y.

$$f'(x) = \frac{1}{2} x^{-1/2}$$

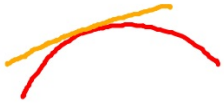
$$f''(x) = \frac{-1}{4} x^{-3/2}$$

$$f''(81) = \frac{-1}{4} \cdot 81^{-3/2}$$

$$f''(81) < 0$$

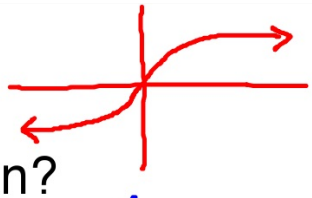
Over/Under Approximmation?

If the funciton is concave down at the point, it will be an over approximation



If the function is concave up at the point, it will be an under approximation

Approximate the cube root of -28.



Is this an over or under approximation?

$f(x) = \sqrt[3]{x} = x^{1/3}$ $\quad (-27, -3) \to (-28, \_\_)$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(-27) = \frac{1}{3}(-27)^{-2/3}$

$= \frac{1}{3} \cdot \frac{1}{9}$

$= \frac{1}{27}$

$y + 3 = \frac{1}{27}(x + 27)$

$y = \frac{1}{27}(-28 + 27) - 3$

$y = \frac{1}{27}(-1) - 3 = -3\frac{1}{27}$

under

$f''(-27) > 0$

ccu

$$f'(x) = \frac{1}{3} x^{-2/3}$$

$$f''(x) = -\frac{2}{9} x^{-5/3}$$

$$f''(-27) = -\frac{2}{9}(-27)^{-5/3}$$

$$f''(-27) > 0$$