3.7 Optimization Problems

<u>Steps to follow</u>

- Solve applied minimum and maximum problems.

1) Write a primary and a secondary equation
Primary: what you are trying to optimize
Secondary: additional information

2) Substitute the secondary information and take the derivative of the primary equation

3) Re-read the question to make sure you have answered the question

#1 Find two positive numbers that satisfy the given requirements.

The sum of the first number squared and the second number is 54 and the product is a maximum.

Primary

$P = xy$

$P = x(54 - x^2)$

$P = 54x - x^3$

$P' = 54 - 3x^2$

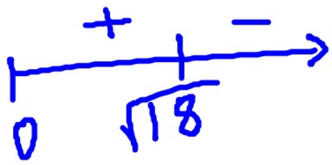
$0 = 54 - 3x^2$

$x = \pm\sqrt{18}$

Secondary

$x^2 + y = 54$

$y = 54 - x^2$



$\boxed{\sqrt{18}, 36}$

$x \quad y$

#2: Find the length and width of a rectangle that has a perimeter of 50 cm maximizes the area.

Primary

$A = xy$

$A = x(25 - x)$

$A = 25x - x^2$

$A' = 25 - 2x$

$\frac{25}{2} = x$

Secondary

$2x + 2y = 50$

$x + y = 25$

$y = 25 - x$

$\boxed{\frac{25}{2}\text{ cm}, \frac{25}{2}\text{ cm}}$

#5: A rectangular page is to contain 24 square inches of print. The margins on the top and bottom are to be 1.5 inches and the margins on the left and right are to be 1 inch. what should be the dimensions of the page so that the least amount of paper is used.

Primary

$A = (x+2)(y+3)$

$A = \left(\frac{24}{y}+2\right)(y+3)$

$A = 24 + \frac{72}{y} + 2y + 6$

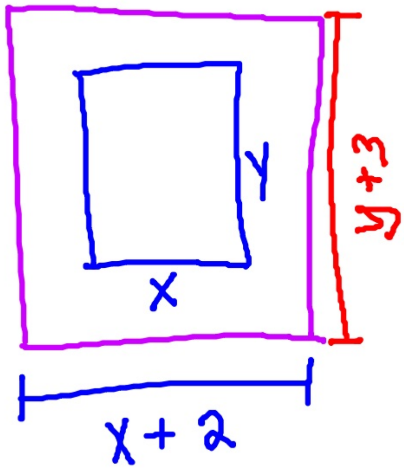
$A = 24 + 72y^{-1} + 2y + 6$

$A' = -72y^{-2} + 2$

Secondary

$xy = 24$

$x = \frac{24}{y}$



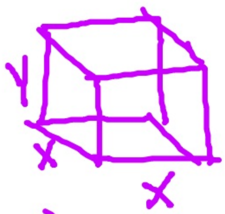
$A' = \frac{-72}{y^2} + 2$

$y = \pm 6$

$\boxed{9\text{in}, 6\text{in}}$

#3 A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a maximum volume?

Primary



$$V = x^2 y$$

$$V = x^2\left(\frac{108 - x^2}{4x}\right)$$

$$V = 27x - \frac{1}{4}x^3$$

$$V' = 27 - \frac{3}{4}x^2$$

$$x = 6$$

Secondary

$$SA = x^2 + 4xy$$

$$108 = x^2 + 4xy$$

$$\frac{108 - x^2}{4x} = y$$

$$\boxed{6in, 6in, 3in}$$

#4: Two pens are to be built alongside a barn as shown.The barn will make up one side of each pen. If 200 ft of fencing are available,what pen size maximizes the area?

Primary

$A = 2xy$

Secondary

$200 = 3x + 2y$

$\boxed{x = \frac{100}{3}\text{ ft} \quad y = 50\text{ ft}}$