3.5 Limits at Infinity

- Determine (finite) limits at infinity.
- Determine the horizontal asymptotes, if any, of the graph of a function.
- Determine infinite limits at infinity.

Limits at Infinity

This section discusses the "end behavior" of a function on an *infinite* interval.

Horizontal Asymptotes

In Figure 3.34, the graph of f approaches the line $y = L$ as x increases without bound. The line $y = L$ is called a **horizontal asymptote** of the graph of f.

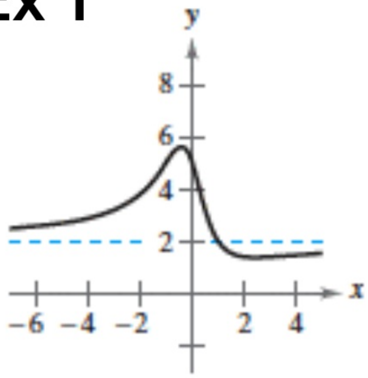
DEFINITION OF A HORIZONTAL ASYMPTOTE

The line $y = L$ is a **horizontal asymptote** of the graph of f if

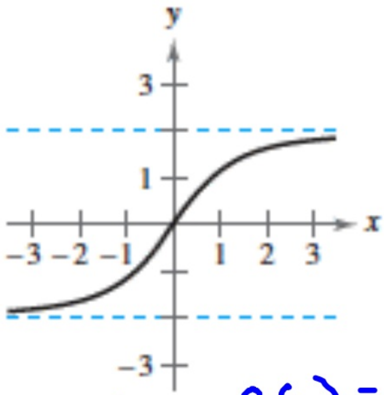
$$\lim_{x\to -\infty} f(x) = L \quad \text{or} \quad \lim_{x\to \infty} f(x) = L.$$

Find the limit as x approaches infinity and negative infinity

Ex 1



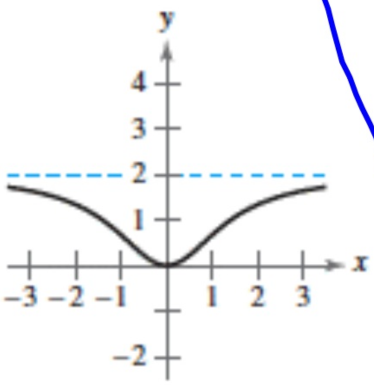
Ex 2



$$\lim_{x\to\infty} f(x) = 2$$

$$\lim_{x\to-\infty} f(x) = -2$$

Ex 3



Ex 4

(a) $\lim_{x\to\infty} \frac{3-2x}{3x^3-1} = 0$

(b) $\lim_{x\to\infty} \frac{3-2x}{3x-1} = -\frac{2}{3}$

(c) $\lim_{x\to\infty} \frac{3-2x^{2}}{3x-1}$ $-\infty$

1000   1000

Both

∞ or $-\infty$

HA?

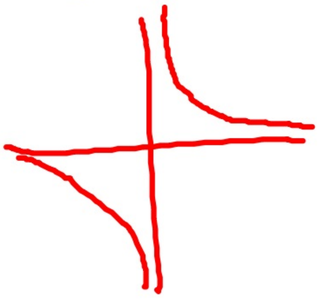
$y=0$

$y=-\frac{2}{3}$

none

Bobo

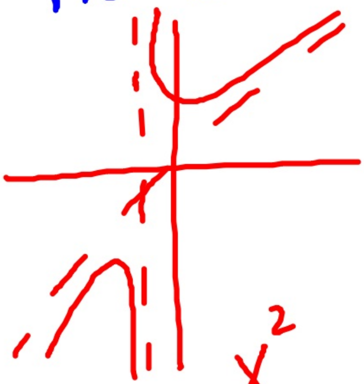
bigger
on
bottom
0



$\frac{1}{x}$

Botn

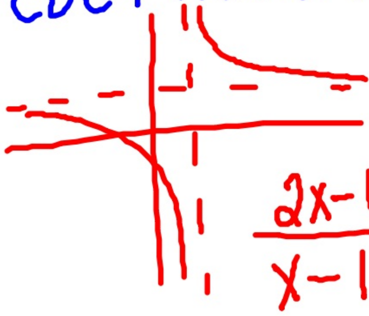
bigger
on
top
none



$\frac{x^2}{x+1}$

eats dc

exponents
are
the
same
divide
coefficients



$\frac{2x-1}{x-1}$

Find the limit.

Ex 5

$$\lim_{x\to-\infty}\left(\frac{5}{x}^{\cdot 3}-\frac{x}{3}^{\cdot +}\right)=\lim_{x\to-\infty}\left(\frac{15-\overset{-1000}{x^2}}{\underset{-1000}{3x}}\right)=\infty$$

$$\frac{\text{neg}}{\text{neg}}$$

Ex 6

$$\lim_{x\to-\infty}\left(\frac{1}{2}x-\frac{4}{x^2}\right)=\lim_{x\to-\infty}\frac{x}{2}-\lim_{x\to-\infty}\frac{4}{x^2}$$

$$-\infty \quad - \quad 0$$

$$-\infty$$

Ex 7

$$\lim_{x\to\infty} \sin x$$

dne

Ex 8

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

$\frac{1}{-10000}$

Ex 9

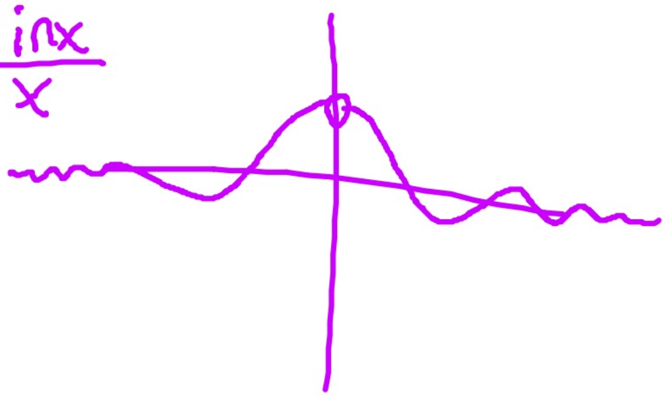
$$\lim_{x\to\infty} \cos\frac{1}{x}$$

$\cos 0$

1

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$\frac{\sin x}{x}$



$$\lim_{x \to \infty} \frac{x}{\sqrt{5x^2+1}}$$

$$\frac{1}{\sqrt{5}}$$

$$\lim_{x \to -\infty} \frac{4x}{\sqrt{9x^2+13}}$$

$$-\frac{4}{\sqrt{9}} = -\frac{4}{3}$$

$$\lim_{x \to -\infty} \frac{-x}{\sqrt{4x^2+7}}$$

$$\frac{1}{\sqrt{4}} = \frac{1}{2}$$

p. 205

15 - 37 odd