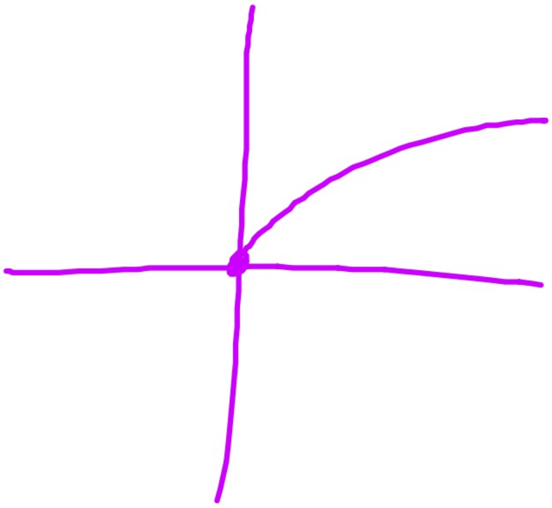
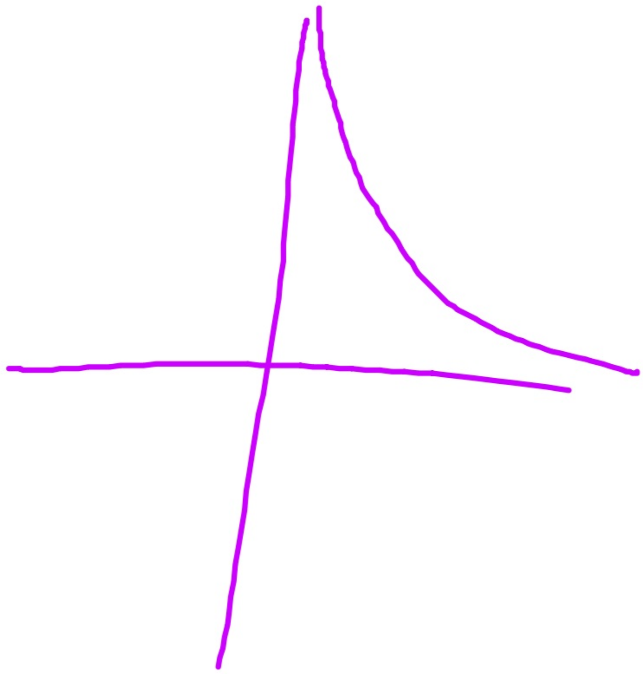
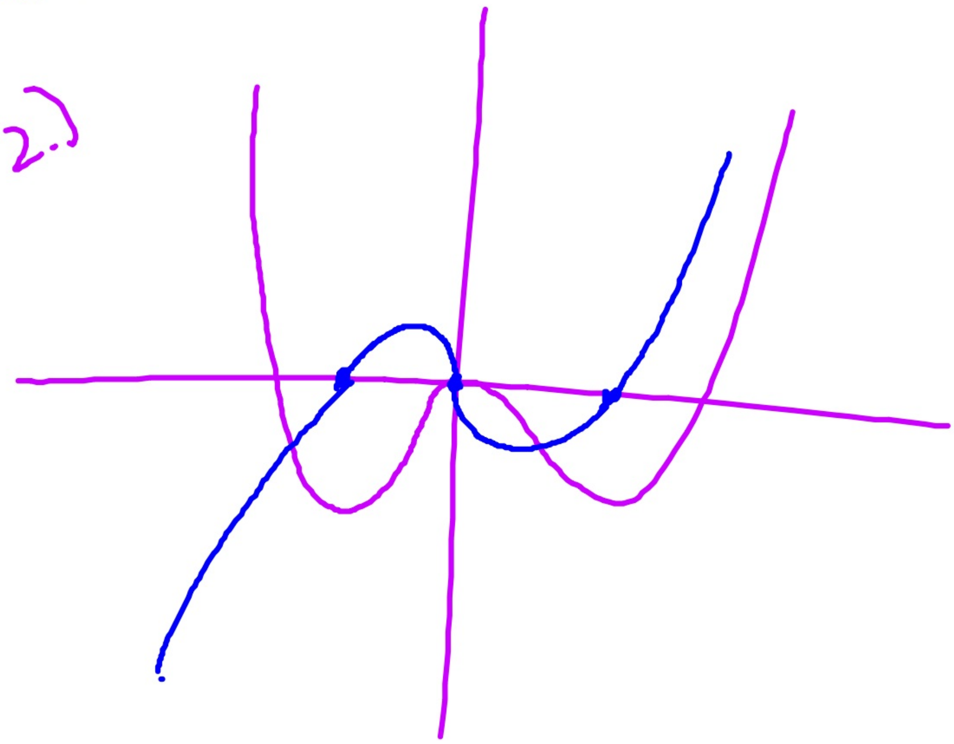
41.) p. 104





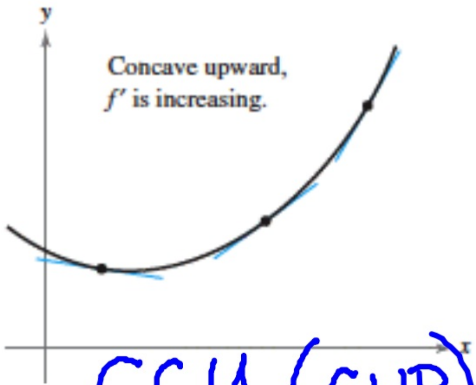
p. 187

(62.)

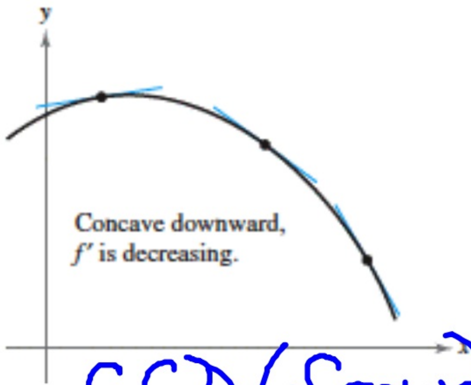


3.4 Concavity and the Second Derivative Test

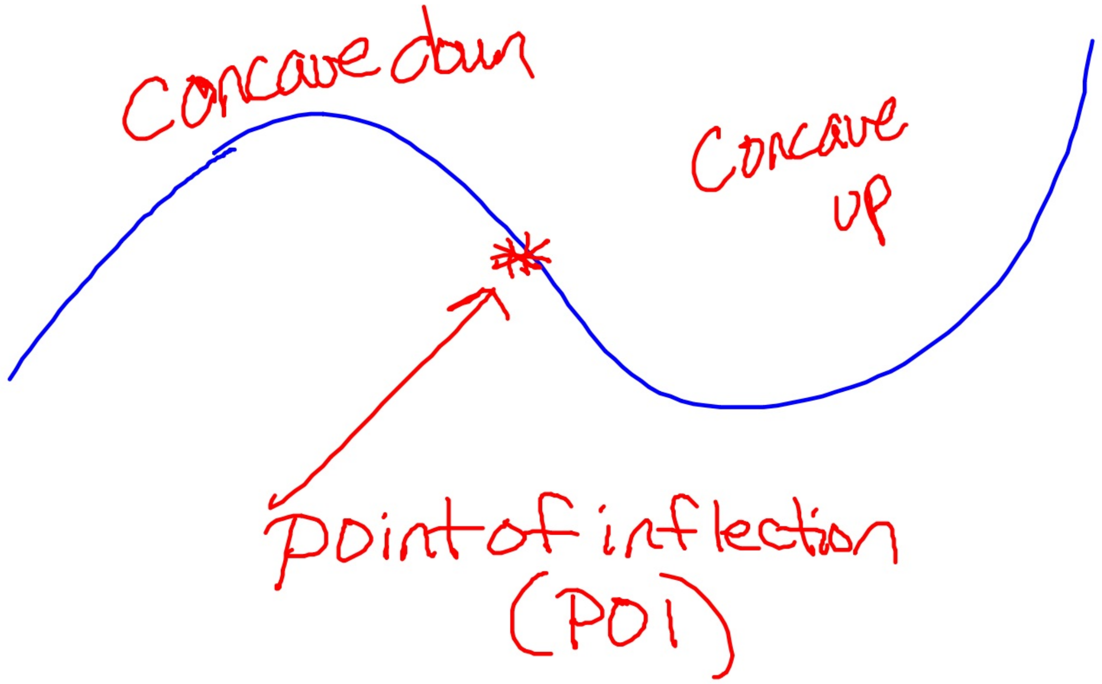
- Determine intervals on which a function is concave upward or concave downward.
- Find any points of inflection of the graph of a function.
- Apply the Second Derivative Test to find relative extrema of a function.



CCU (cup)

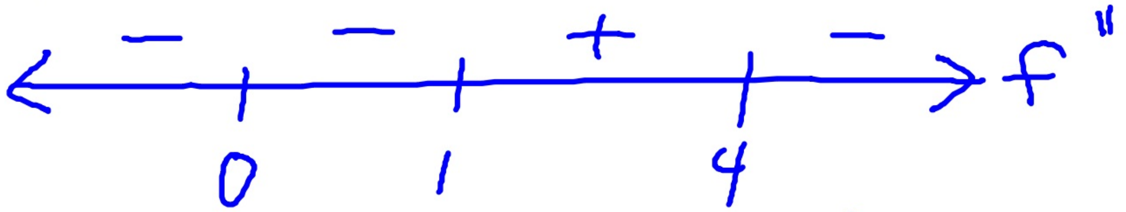


CCD (frown)


Concave down
Concave up
point of inflection
(POI)

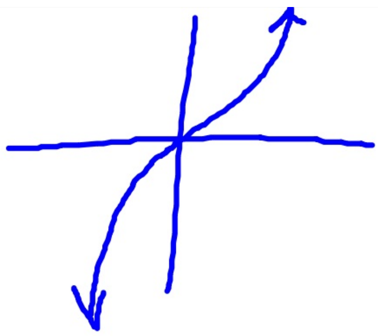
3.8 POINTS OF INFLECTION

is a point of inflection of the graph of f, then either $f''(c) = 0$ or
xist at $x = c$.



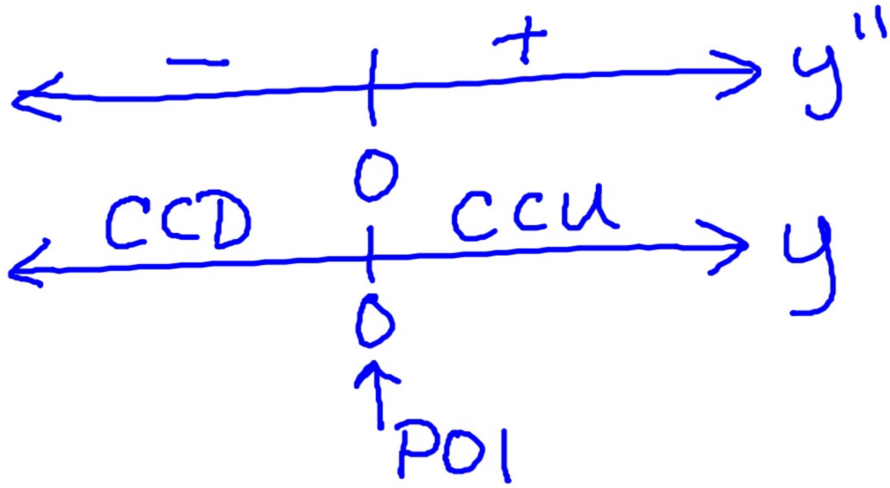
$x = 1, 4$ POI because f'' changes signs at these values

$y = x^3$



$y' = 3x^2$

$y'' = 6x$



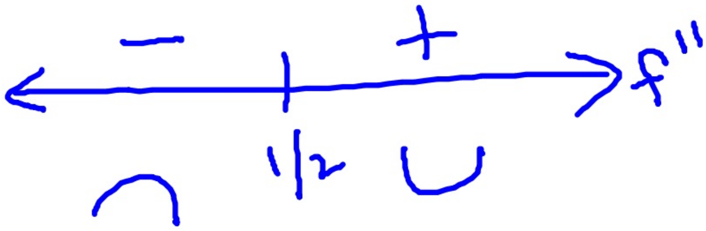
Find the intervals where f is concave up or down.
Also, find the x-value of point(s) of inflection. $D: (-\infty, \infty)$

#1

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$

$$f'(x) = 6x^2 - 6x - 12$$

$$f''(x) = 12x - 6$$



$$0 = 12x - 6$$

$$1/2 = x$$

POI at x = 1/2 because f'' changes signs at this valu

CCD on (-inf,1/2) because f''<0 on this interva

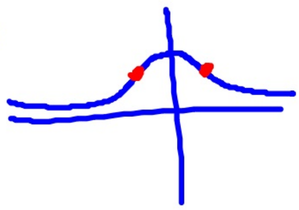
CCU on (1/2, inf) because f''>0 on this interva

#2 $g(x) = \frac{4}{x^2+1} = 4(x^2+1)^{-1}$ D: $(-\infty, \infty)$

$g'(x) = -8x(x^2+1)^{-2}$

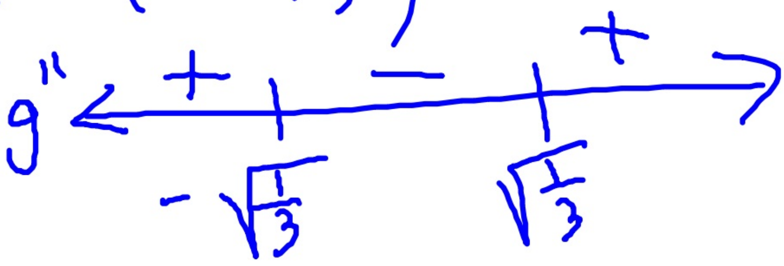
$g''(x) = -8x\left(-2(x^2+1)^{-3} \cdot 2x\right) + (x^2+1)^{-2} \cdot -8$

$= 32x^2(x^2+1)^{-3} + -8(x^2+1)^{-2}$



$= 8(x^2+1)^{-3}\left(4x^2 - (x^2+1)^1\right)$

$= \frac{8(3x^2-1)}{(x^2+1)^3}$



We use the first derivative to find:

intervals of increasing/decreasing and

relative extrema

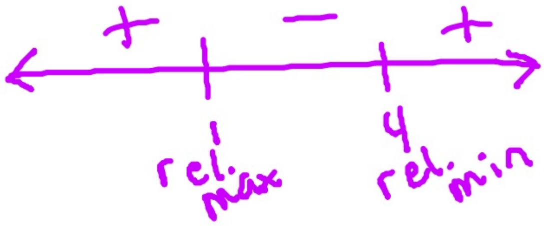
We use the second derivative to find:

intervals of concavity and

points of inflection

But, there are two tests that have an official name

First Derivative Test



Second Derivative Test

BOTH tests are ways to locate relative extrema

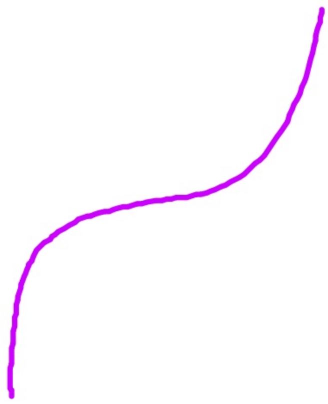
local

THEOREM 3.9 SECOND DERIVATIVE TEST

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

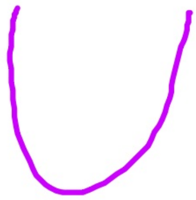
1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$. CCU
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$. CCD

If $f''(c) = 0$, the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.



f'' = 0

Neither



f'' > 0
CCU

Rel min

f'' < 0
CCD

Rel max

$f'(4) = 0$

$f''(4) = -8$

$f(4)$ rel. max

CCD

$f'(-3) = 0$

$f''(-3) = 12$

$f(-3)$ rel. min

CCU

Find the relative extrema using the second derivative test.

#1 $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2$

$0 = -15x^2(x^2 - 1)$

$x = 0, 1, -1$

$f''(x) = -60x^3 + 30x$

$f''(1) < 0$; rel. max $(1, 2)$

$f''(-1) > 0$; rel. min $(-1, -2)$

$f''(0) = 0$; neither $(0, 0)$

Find the x-coordinates of the relative extrema

#2

$$f(x) = x^3 - 5x^2 + 7x$$

$$f'(x) = 3x^2 - 10x + 7$$

$$0 = (3x - 7)(x - 1)$$

$$x = 7/3,\ 1$$

$$f''(x) = 6x - 10$$

$f''(7/3) > 0$; $\cup$ rel. min at $x = 7/3$

$f''(1) < 0$; $\cap$ rel. max at $x = 1$

Sketching a derivative (and second derivative)

Finding intervals of concavity

Finding points of inflection

Using the second derivative test

11.) $f(x) = \frac{24}{x^2+12} = 24(x^2+12)^{-1}$

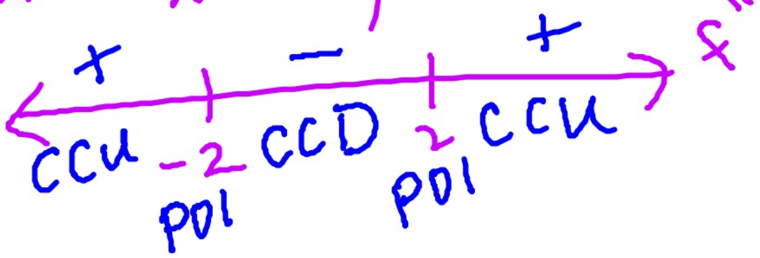
$f'(x) = -24(x^2+12)^{-2}\cdot 2x = -48x(x^2+12)^{-2}$

$f''(x) = -48\left(x\cdot -2(x^2+12)^{-3}\cdot 2x + (x^2+12)^{-2}\cdot 1\right)$

$= -48\left(-4x^2(x^2+12)^{-3} + (x^2+12)^{-2}\right)$

$= -48\left((x^2+12)^{-3}(-4x^2 + x^2 + 12)\right)$

$= \frac{-48(12-3x^2)}{(x^2+12)^3}$



5.) $f(x) = x^2 - x - 2$ $\qquad \cup$

$f'(x) = 2x - 1$

$f''(x) = 2 \qquad \xleftrightarrow{\quad + \quad} f''$

No POI

CCU $(-\infty, \infty)$