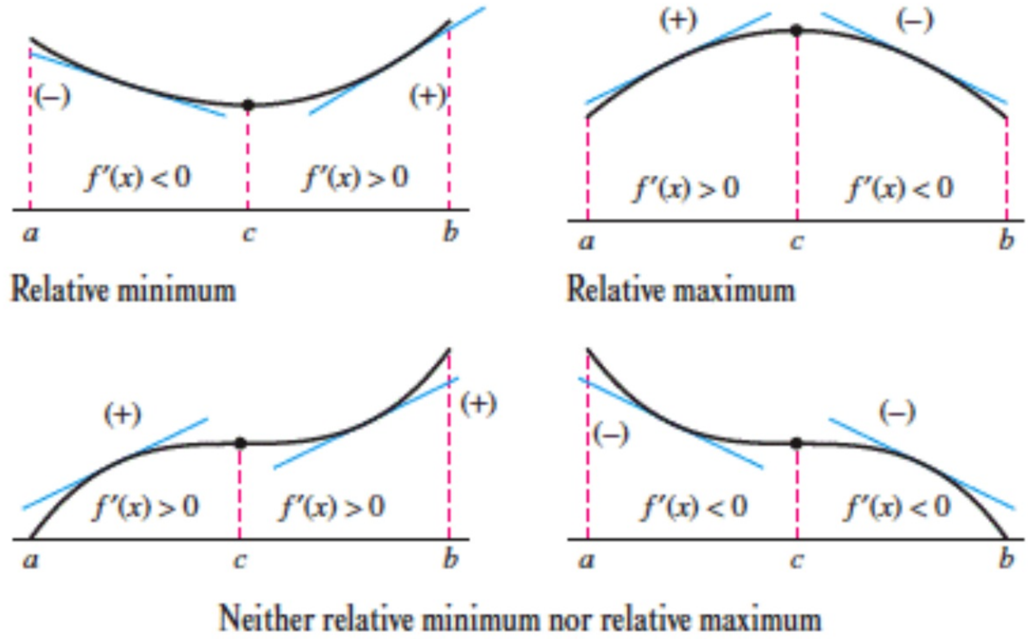


The First Derivative Test

points

#1 Find the relative extrema. Justify your answer.

$f(x)=x^3 - 6x^2$ $D: (-\infty, \infty)$ $f(4) = 64 - 6 \cdot 16$

$f'(x) = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, 4$

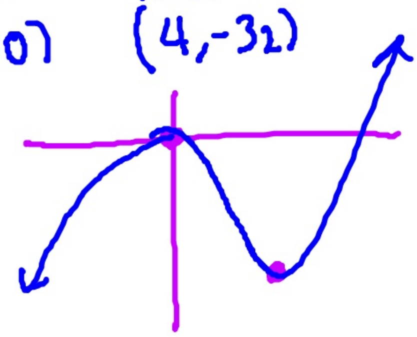
$f'(x)$: + | 0 | − | 4 | +

$f(x)$: incr. | 0 | decr. | 4 | incr.

rel. max (0, 0)   rel. min (4, -32)

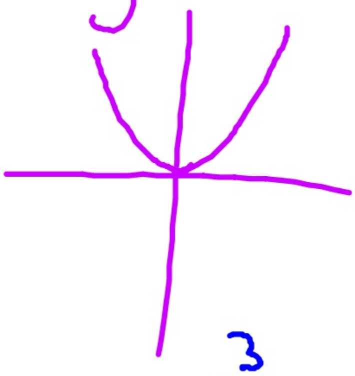
Relative max at (0,0) because f' changes signs from positive to negative at this point.

Relative min at (4, -32) because f' changes signs from negative to positive at this point.

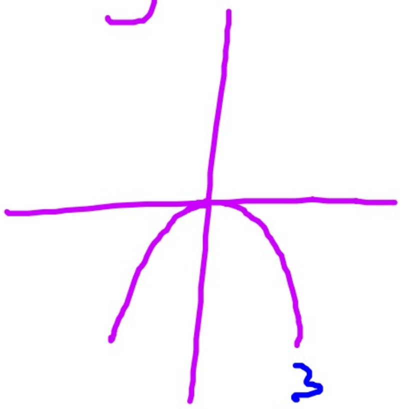


end behavior of polynomials

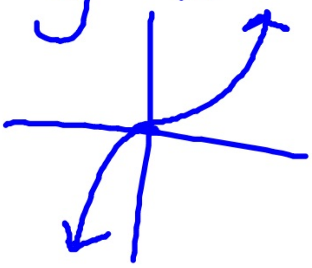
$y = x^2$



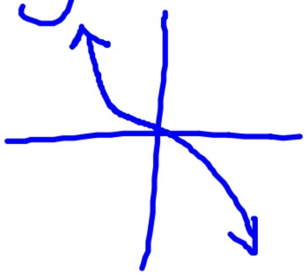
$y = -x^2$



$y = x^3$



$y = -x^3$



Find all relative extrema.

#2

$$f(x) = (x^2 - 4)^{2/3} \qquad D: (-\infty, \infty)$$

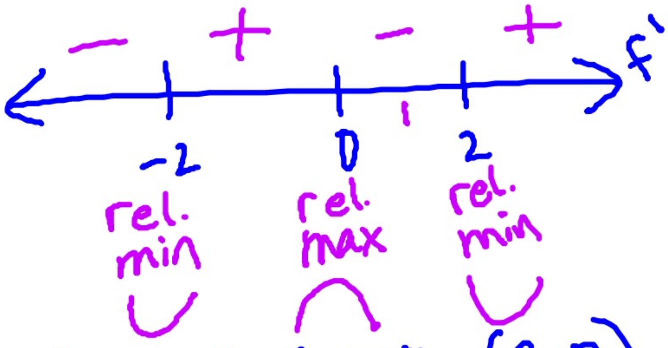
$$(0-4)^{2/3} = 4^{2/3}$$

$$f'(x) = \frac{2}{3}(x^2-4)^{-1/3} \cdot 2x$$

$$f'(x) = \frac{4x^1}{3\sqrt[3]{x^2-4}}$$

$$4x = 0 \qquad 3\sqrt[3]{x^2-4} = 0$$

$$x = 0 \qquad x = \pm 2$$



$(-2, 0)$ $(0, \sqrt[3]{16})$ $(2, 0)$

There is a relative min at (-2, 0) and (2,0) because f' changes from negative to positive at these points.

There is a relative max at $(0, \sqrt[3]{16})$ because f' changes from positive to negative at this point.

#3 Find the x-values of the relative extrema.

$D: \{x \mid x \neq 2\}$

VA @ $x = 2$

$$f(x) = \frac{x^2 - 2x + 4}{x - 2}$$

$$f'(x) = \frac{(x-2)(2x-2) - (x^2 - 2x + 4)}{(x-2)^2}$$

$$f'(x) = \frac{2x^2 - 6x + 4 - x^2 + 2x - 4}{(x-2)^2}$$

$$f'(x) = \frac{x(x-4)}{(x-2)^2}$$

$x = 0, 4$