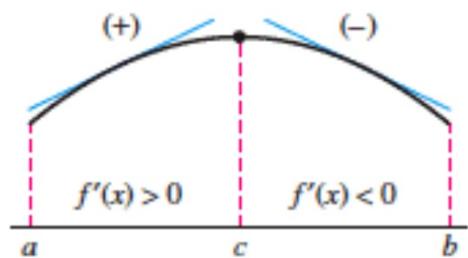
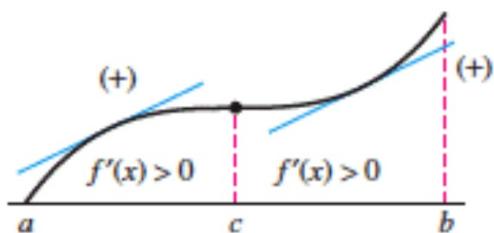


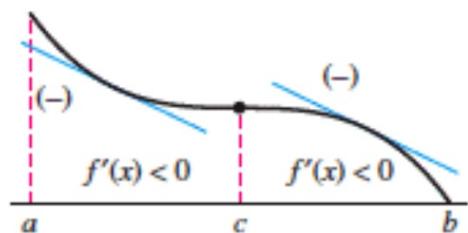
Relative minimum



Relative maximum



Neither relative minimum nor relative maximum



The First Derivative Test

#1 Find the relative extrema. Justify your answer.

$f(x) = x^3 - 6x^2$ $D: (-\infty, \infty)$ $f(4) = 64 - 6 \cdot 16$

$f'(x) = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, 4$

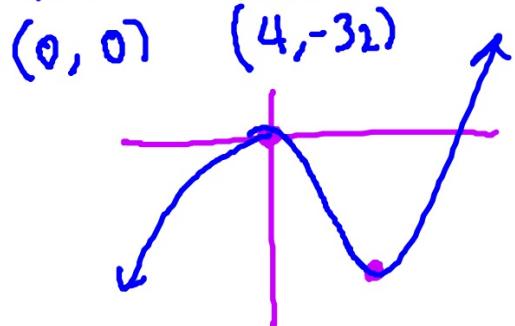
Relative max at (0,0) because f' changes signs from positive to negative at this point.

The sign chart for $f'(x)$ shows the following intervals:

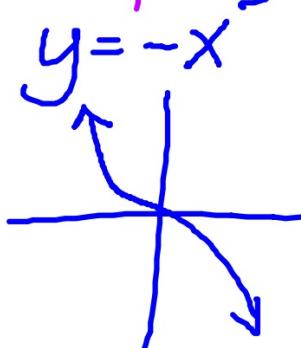
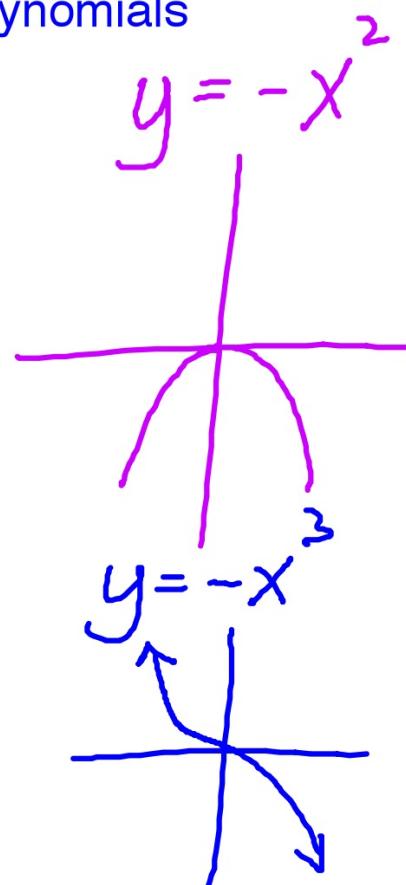
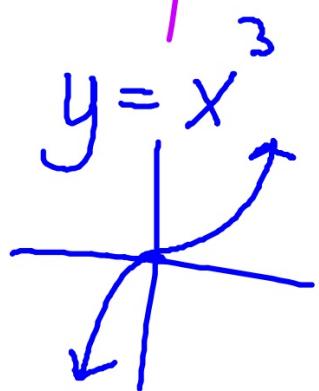
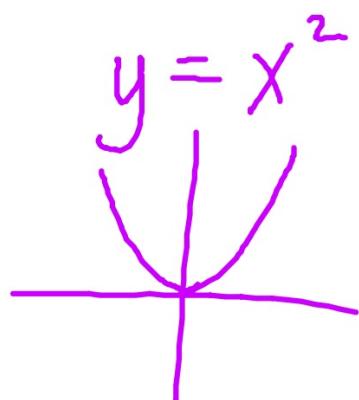
- For $x < 0$, $f'(x) > 0$ (increasing, labeled "inc.")
- At $x = 0$, $f'(x) = 0$ (critical point)
- For $0 < x < 4$, $f'(x) < 0$ (decreasing, labeled "decr.")
- At $x = 4$, $f'(x) = 0$ (critical point)
- For $x > 4$, $f'(x) > 0$ (increasing, labeled "inc.")

Relative min at (4, -32) because f' changes signs from negative to positive at this point.

Relative min at (4, -32) because f' changes signs from negative to positive at this point.



end behavior of polynomials



Find all relative extrema.

#2 $f(x) = (x^2 - 4)^{2/3}$ $D: (-\infty, \infty)$

$$f'(x) = \frac{2}{3}(x^2 - 4)^{-1/3} \cdot 2x$$
$$f'(x) = \frac{4x}{3\sqrt[3]{x^2 - 4}}$$
$$4x = 0 \quad 3\sqrt[3]{x^2 - 4} = 0$$
$$x = 0 \quad x = \pm 2$$

The sign chart shows the derivative f' on a number line with points $-2, 0, 2$. The intervals are labeled with signs: $-$, $+$, $-$, $+$. Below the line, the points $-2, 0, 2$ are marked with vertical lines. To the left of -2 , there is a label "rel. min" with a U-shaped arrow below the axis. Between -2 and 0 , there is a label "rel. max" with a U-shaped arrow below the axis. To the right of 2 , there is a label "rel. min" with a U-shaped arrow below the axis. Below the axis, the points $(-2, 0)$, $(0, \sqrt[3]{16})$, and $(2, 0)$ are written.

There is a relative min at $(-2, 0)$ and $(2, 0)$ because f' changes from negative to positive at these points.

There is a relative max at $(0, \sqrt[3]{16})$ because f' changes from positive to negative at this point.

#3 Find the x-values of the relative extrema.

$$D: \{x \mid x \neq 2\}$$

$$\text{VA} @ x = 2$$

$$f(x) = \frac{x^2 - 2x + 4}{x - 2}$$

$$f'(x) = \frac{(x-2)(2x-2) - (x^2 - 2x + 4)}{(x-2)^2}$$

$$f'(x) = \frac{2x^2 - 6x + 4 - x^2 + 2x - 4}{(x-2)^2}$$

$$f'(x) = \frac{x(x-4)}{(x-2)^2}$$

$$x=0, 4$$

