3.3 Increasing and Decreasing Functions and the First Derivative Test

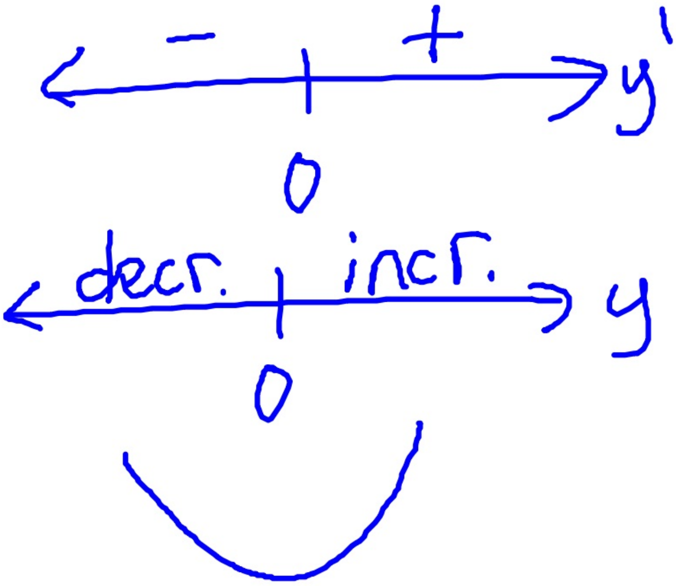
- Determine intervals on which a function is increasing or decreasing.
- Apply the First Derivative Test to find relative extrema of a function.

THEOREM 3.5 TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

$$y = x^2$$
$$y' = 2x$$
$$0 = 2x$$
$$0 = x$$



Find the intervals where the function is increasing or decreasing. Justify your answer.

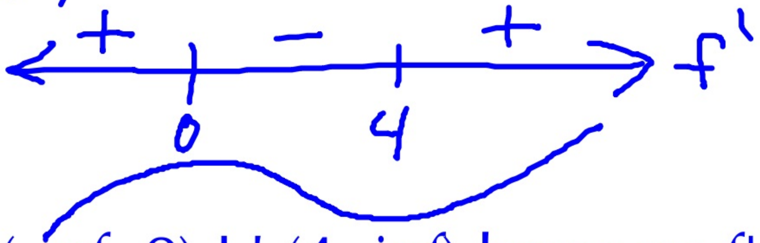
#1 $f(x) = x^3 - 6x^2 + 15$ $D: (-\infty, \infty)$

$f'(x) = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, 4$

Find the critical numbers.



f is increasing on (-inf, 0) U (4, inf) because f'>0 on these intervals

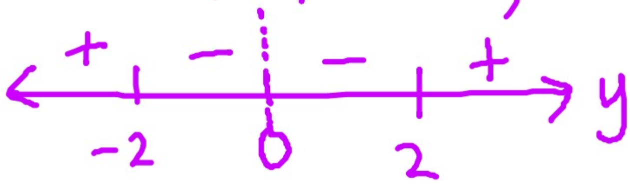
f is decreasing on (0,4) because f'<0 on this interval

Find the intervals where the function is increasing or decreasing. Justify.

$D: \{x \mid x \neq 0\}$

#2

$$y = x + \frac{4}{x}$$



$y = x + 4x^{-1}$

$y' = 1 - 4x^{-2}$

$y' = 1 - \frac{4}{x^2}$

$y' = \frac{x^2 - 4}{x^2}$

$x = \pm 2$

y is increasing on (-inf,-2) U (2, inf) because f'>0 on these intervals

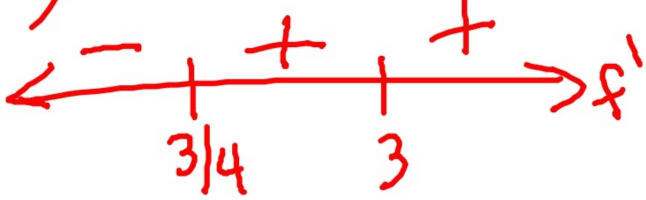
y is decreasing on (-2, 0) U(0,2) because f' < 0 on these intervals

3) $f(x) = x(3x-9)^3$ D: $(-\infty, \infty)$

$f'(x) = \underbrace{x \cdot 3(3x-9)^2 \cdot 3} + \underbrace{(3x-9)^3 \cdot 1}$

$= (3x-9)^2(9x+3x-9)$

$= (3x-9)^2(12x-9)^1$



$x = 3, \ 3/4$

Increasing: (3/4,3)
U (3, inf)

Decreasing: (-inf,3/4)

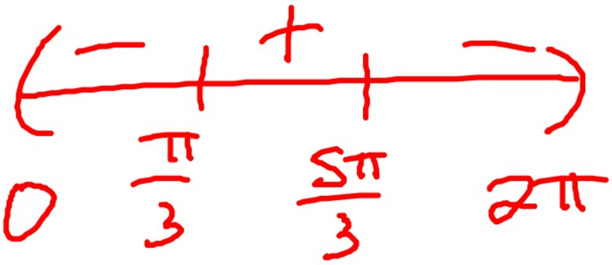
4) $f(x) = x - 2\sin x \quad (0, 2\pi)$

$f'(x) = 1 - 2\cos x$

$0 = (1 - 2\cos x)$

$\frac{1}{2} = \cos x$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$



p.196: 5-15 odd

5,7: look at graph

11: D: [-4,4]