3.2 Rolle's Theorem and the Mean Value Theorem

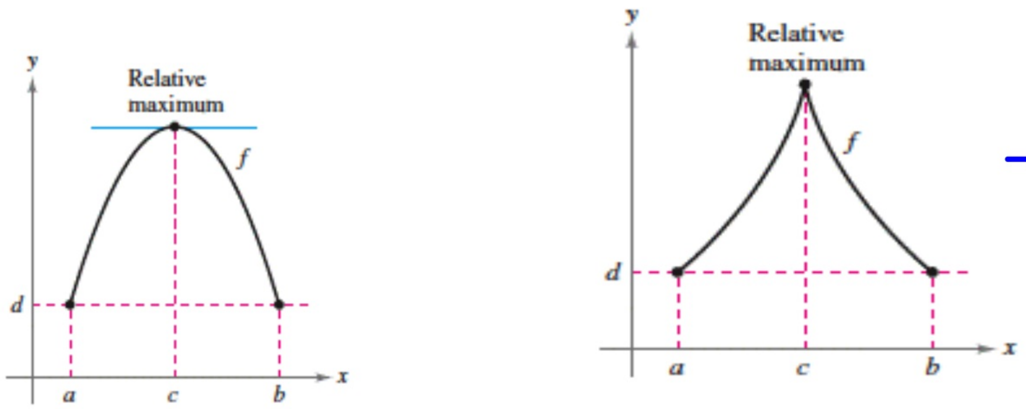
- Understand and use Rolle's Theorem.
- Understand and use the Mean Value Theorem.

3.3 ROLLE'S THEOREM

ntinuous on the closed interval $[a, b]$ and differentiable on the op
, b). If

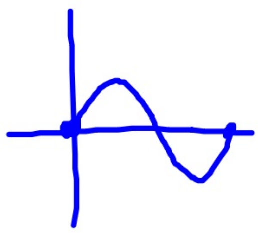
= $f(b)$

is at least one number c in (a, b) such that $\boxed{f'(c) = 0.}$



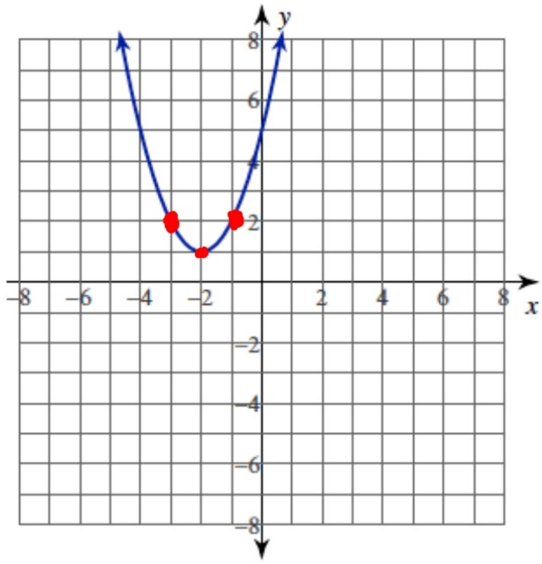
(a) f is continuous on $[a, b]$ and differentiable on (a, b).

(b) f is continuous on $[a, b]$.



#1 Verify if Rolle's Theorem applies. Then find the value(s) where f'(c) = 0

$y = x^2 + 4x + 5; \quad [-3, -1]$



cont. $[-3, -1]$ ✓

diff $(-3, -1)$ ✓

$f(-3) = 2$

$f(-1) = 2$

$y' = 2x + 4$

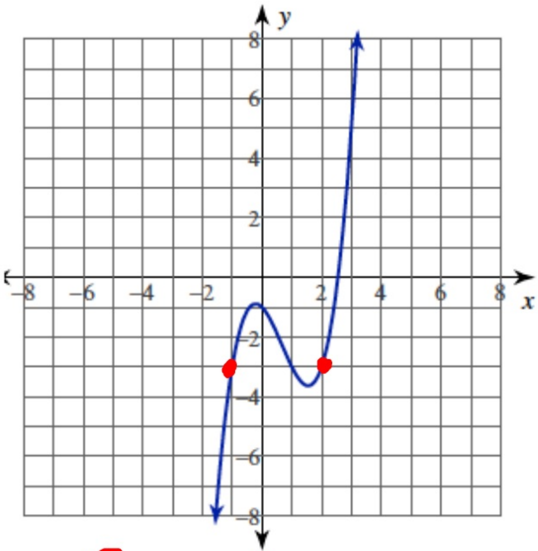
$2x + 4 = 0$

$\boxed{x = -2}$

#2: Verify if Rolle's Theorem applies.

Then find the value(s) where f'(c) = 0

$y = x^3 - 2x^2 - x - 1;\quad [-1, 2]$



CONT $[-1, 2]$ ✓

diff $(-1, 2)$ ✓

$f(-1) = f(2) = -3$

$3x^2 - 4x - 1 = 0$

$x = \frac{4 \pm 2\sqrt{7}}{6}$

$\boxed{x = \frac{2 \pm \sqrt{7}}{3}}$

$y = x^3 - 2x^2 - x - 1$

#5: Verify if Rolle's Theorem applies.

Then find the value(s) where f'(c) = 0

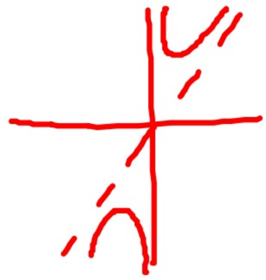
$$y = \frac{-x^2 + 2x + 15}{x + 4}; \quad [-3, 5]$$

cont. $[-3, 5]$ ✓

diff. $(-3, 5)$ ✓

$f(-3) = 0$

$f(5) = 0$



$$\cancel{f'(x)} = \frac{-(x^2 + 8x + 7)}{(x+4)^2}$$

$$0$$

$$0 = -(x+7)(x+1)$$

$$x = -1, \cancel{-7}$$

#3: Verify if Rolle's Theorem applies.
Then find the value(s) where f'(c) = 0

$f(x) = 3 - |x - 3|, \quad [0, 6]$

Cont. [0, 6] ✓
diff. (0, 6) ✗
corner at x = 3
Rolle's theorem does not apply.

#4: Verify if Rolle's Theorem applies.

Then find the value(s) where f'(c) = 0

$$y = \frac{-x^2 + 4}{4x}; \quad [-2, 2]$$

Cont. $[-2, 2]$ X

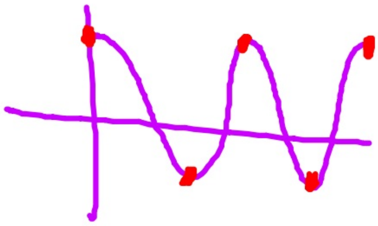
$x = 0$; VA

Rolle's does not apply.

#6: Verify if Rolle's Theorem applies.

Then find the value(s) where f'(c) = 0

$$f(x) = \cos 2x, \quad [0, 2\pi]$$



cont. $[0, 2\pi]$ ✓

diff. $(0, 2\pi)$ ✓

$f(0) = 1$ ✓

$f(2\pi) = 1$

$f'(x) = -2\sin 2x$

$0 = -2\sin 2x$

$0 = \sin 2x$ $\quad 2x = A$

$0 = \sin A$

$\frac{0}{2}, \frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}, \frac{4\pi}{2} = 2x$

$\cancel{0}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \cancel{2\pi} = x$

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11 14 15 18

19 24 25 26