3.2 Rolle's Theorem and the Mean Value Theorem

- Understand and use Rolle's Theorem.
- Understand and use the Mean Value Theorem.

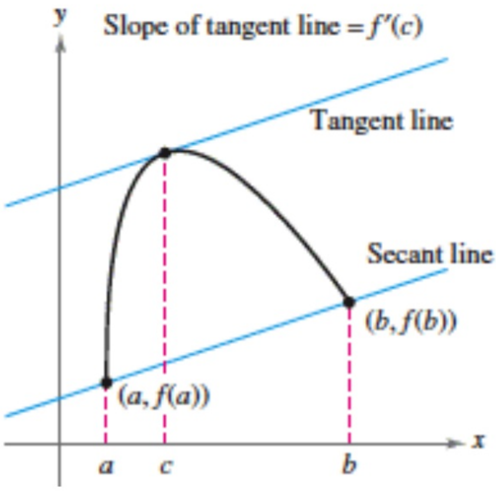
MVT

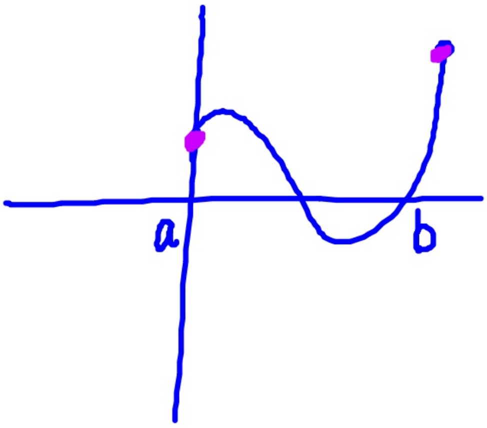
THEOREM 3.4 THE MEAN VALUE THEOREM

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

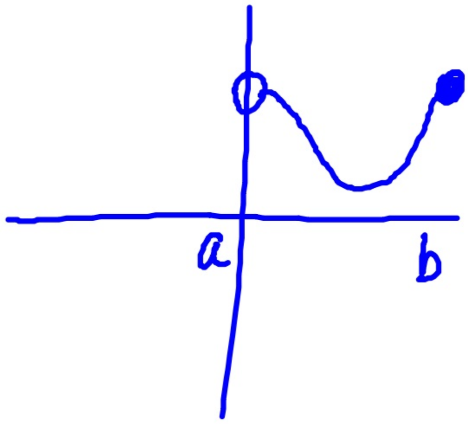
$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$M_{tan} = M_{sec}$





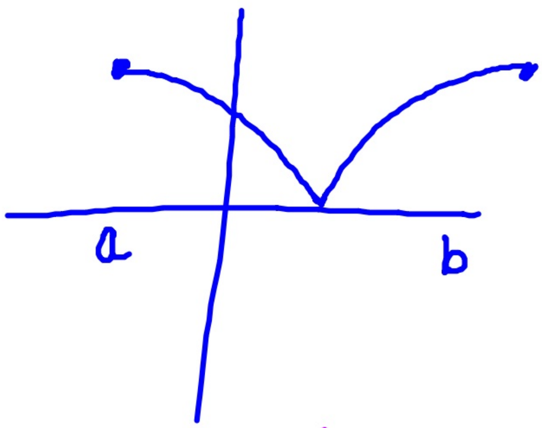
cont ✓
diff ✓



Cont. X [a, b]

diff. (a, b)

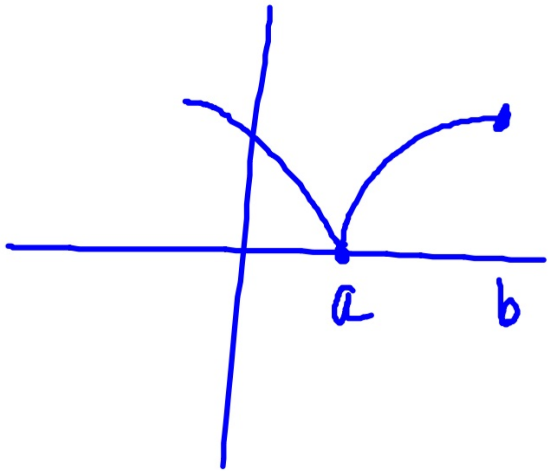
MVT does not apply



Cont. ✓

diff ✗

cusp; MVT does not apply



Cont. $[a,b]$ ✓

diff (a,b) ✓

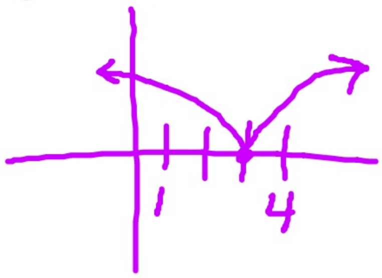
Determine if the Mean Value Theorem applies.

1) $y = -\dfrac{x^2}{4x+8}$; $[-3, -1]$

disc. at $x = -2$

MVT does not apply

2) $y = (x-3)^{\frac{2}{3}}$; $[1, 4]$



cont. $[1,4]$ ✓

diff $(1,4)$ ✗

cusp at $x=3$

MVT does not apply.

3) Verify if MVT Theorem is applicable.

If so, find the value(s) c that satisfy the MVT.

$f(x) = -x^2 + 8x - 17; \quad [3, 6]$

cont. [3, 6] ✓

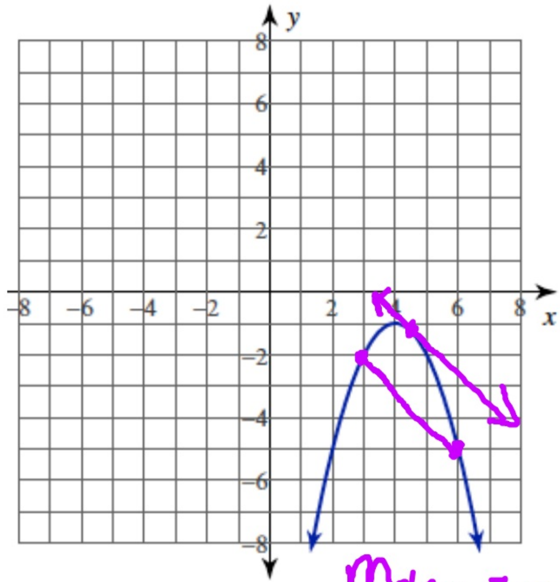
diff (3, 6) ✓

$m_{tan} = m_{sec}$

$-2x + 8 = -1$

$-2x = -9$

$\boxed{x = \frac{9}{2}}$



$m_{sec} = -1$

#4 $y = \dfrac{-x^2+9}{4x}$ [1, 3]

$(1, 2)(3, 0)$

cont. [1, 3] ✓
diff (1, 3) ✓

$m_{sec} = \dfrac{0-2}{3-1} = -1$

$$m_{tan} = y' = \frac{-4x^2-36}{16x^2}$$

$m_{tan} = m_{sec}$

$$\frac{-4x^2-36}{16x^2} = \frac{-1}{1}$$

$$-4x^2-36 = -16x^2$$
$$12x^2 = 36$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$
$$\boxed{x = \sqrt{3}}$$

#5 $y = -x^3 + 4x^2 - 3; \quad [0, 4]$

cont. ✓

diff ✓

$(0, -3)$

$(4, -3)$

$-3x^2 + 8x = 0$

$m_{tan} = m_{sec}$

$-x(3x-8) = 0$

$x = 0, \boxed{\frac{8}{3}}$

#6

$y = x^{1/3}$ on [0, 8]

Cont. [0,8] ✓
diff (0,8) ✓

$m_{tan} = m_{sec}$

$\frac{1}{3}x^{-2/3} = \frac{1}{4}$

$x^{-2/3} = \frac{3}{4}$

$\left(x^{2/3}\right)^{3/2} = \left(\frac{4}{3}\right)^{3/2}$

(0, 0)(8, 2)

$m_{sec} = \frac{1}{4}$

$x = \left(\frac{4}{3}\right)^{3/2} = \pm\sqrt{\frac{64}{27}}$

±

$\boxed{x = \frac{8}{3\sqrt{3}}}$