3.2 Rolle's Theorem and the Mean Value Theorem

- Understand and use Rolle's Theorem.
- Understand and use the Mean Value Theorem.

Chapter 3a Test Topics (Nov 10)

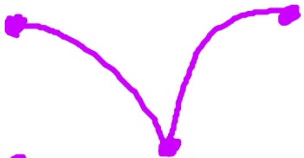
Related Rates
Motion
3.1
3.2

M 3.4 THE MEAN VALUE THEOREM (MVT)

tinuous on the closed interval $[a, b]$ and differentiable on the o
$a, b)$, then there exists a number c in (a, b) such that

$$= \frac{f(b) - f(a)}{b - a}.$$

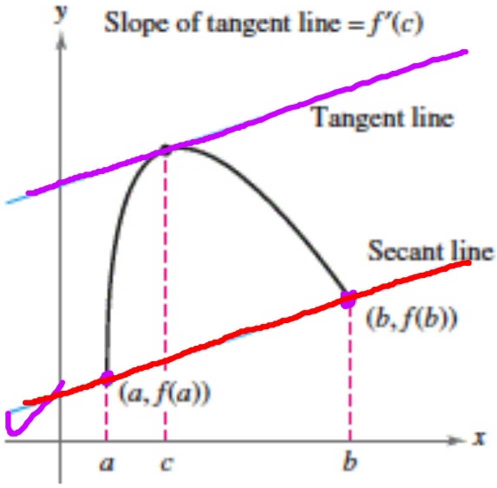
$m_{tan} = m_s$

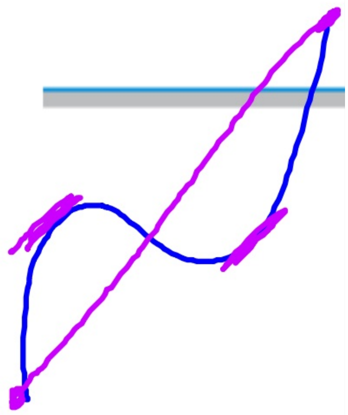


a b c

✓ (a,b): diff.

X (a,c): not diff.





Verify if MVT Theorem is applicable. If so, find the value(s) c that satisfy the MVT.

#1 $f(x) = x^4 - 8x, \quad [0, 2]$

$f'(x) = 4x^3 - 8$

$\frac{f(2)-f(0)}{2-0} = \frac{0-0}{2} = 0$

Verify

$f(x)$ cont. $[0,2]$ ✓

$f(x)$ diff $(0,2)$ ✓

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$4c^3 - 8 = 0$

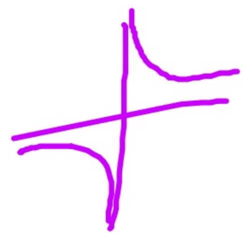
$c^3 = 2$

$\boxed{c = \sqrt[3]{2}}$

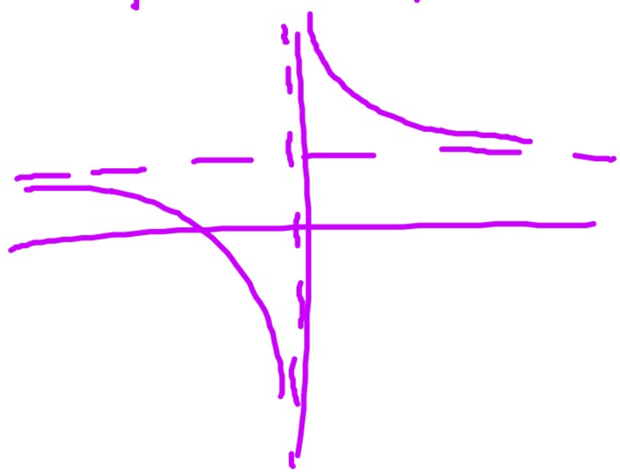
$f'(\sqrt[3]{2}) = 0$

#2 $f(x) = \dfrac{x+1}{x}, \quad [-1, 2]$

$f(x)$ is discontinuous at $x=0$
MVT does not apply.



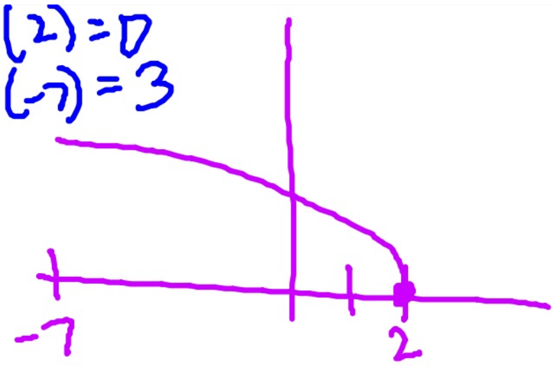
$f(x) = 1 + \dfrac{1}{x}$



#3 $f(x) = \sqrt{2 - x}, \quad [\overset{a}{-7}, \overset{b}{2}]$

$f(2) = 0$
$f(-7) = 3$

Cont. $[-7, 2]$ ✓
diff. $(-7, 2)$ ✓



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{1}{2}(2-x)^{-1/2} \cdot -1 = \frac{0-3}{2--7}$$

$$\frac{+1}{2\sqrt{2-x}} = \frac{+1}{3}$$

$$2\sqrt{2-x} = 3$$

$$\sqrt{2-x} = \frac{3}{2}$$

$$2 - x = \frac{9}{4}$$

$$2 - \frac{9}{4} = x$$

$$-\frac{1}{4} = x$$

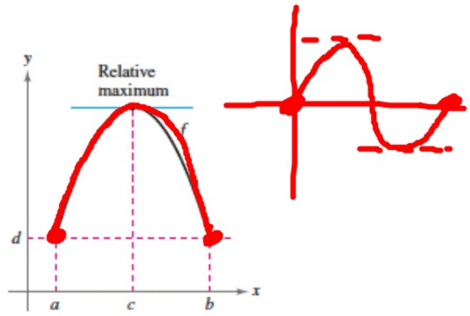
$$c = -\frac{1}{4}$$

THEOREM 3.3 ROLLE'S THEOREM

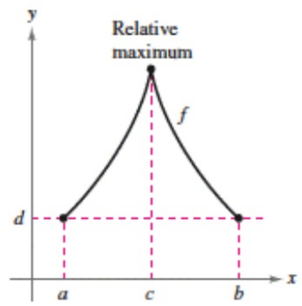
Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If

$$f(a) = f(b)$$

then there is at least one number c in (a, b) such that $f'(c) = 0$.



(a) f is continuous on $[a, b]$ and differentiable on (a, b).



(b) f is continuous on $[a, b]$.

Verify if Rolle's Theorem applies.
Then find the value(s) where
f'(c) = 0

#4 $f(x) = x^2 - 5x + 4, \quad [1, 4]$

Verify

cont. [1,4] ✓
diff (1,4) ✓
$f(1) = f(4)$
$0 = 0$ ✓

$f'(x) = 2x - 5$

$2x - 5 = 0$

$x = \frac{5}{2}$

$c = \frac{5}{2}$

$f'\left(\frac{5}{2}\right) = 0$

Verify if Rolle's Theorem applies.
Then find the value(s) where
f'(c) = 0

#5 $f(x) = 3 - |x - 3|, \quad [0, 6]$

Verify

cont. [0,6] ✓
diff (0,6) ✗
Rolle's does not apply. (f is not diff. at x = 3)

#6

$$f(x) = \frac{x^2 - 1}{x}, \quad [-1, 1]$$

Cont. $[-1, 1]$ X

Rolle's Thm does not apply.

$f(x)$ is disc. at $x = 0$.

#7

$f(x) = \cos 2x, \ [-\pi, \pi]$

Rolle's

Verify

f is cont. $[-\pi, \pi]$ ✓

f is diff $(-\pi, \pi)$ ✓

$f(-\pi) = f(\pi)$

$1 = 1$ ✓

$$f'(x) = -\sin(2x)\cdot 2$$

$$-2\sin 2x = 0$$

$$\sin(2x) = 0$$

$$\frac{2x}{2} = \frac{\pi}{2}, \frac{2\pi}{2}, \frac{0}{2}, \frac{-\pi}{2}, \frac{-2\pi}{2}$$

$$x = \frac{\pi}{2}, \cancel{\pi}, 0, \frac{-\pi}{2}, \cancel{-\pi}$$

$(-\pi, \pi)$

$$c = 0, \frac{\pi}{2}, \frac{-\pi}{2}$$

3.1: Absolute Extrema/relative extrema

critical values

3.2: MVT

Rolle's