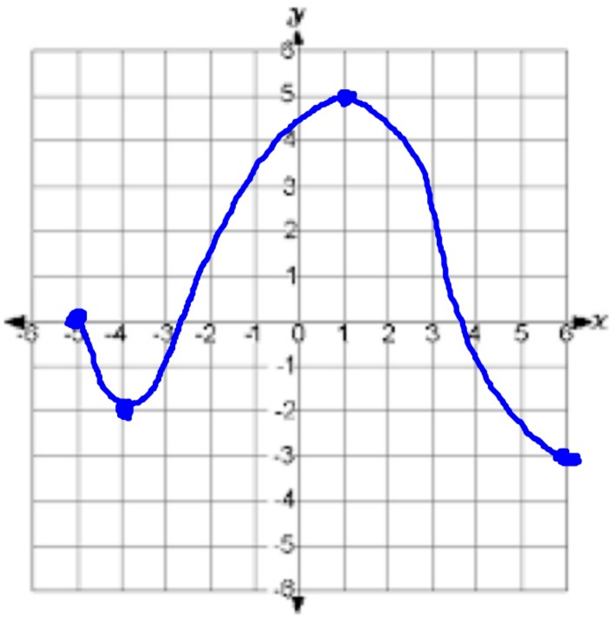
3.1 Extrema on an Interval

- Understand the definition of extrema of a function on an interval.
- Understand the definition of relative extrema of a function on an open interval.
- Find extrema on a closed interval.

Is there an absolute min? max? $[-5, 6]$

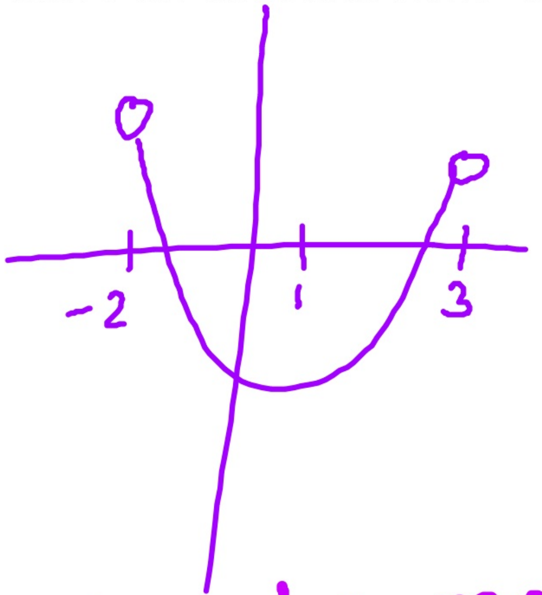


Absolute max: $(1, 5)$

Absolute min: $(6, -3)$

Absolute extrema can be at an endpoint

Is there an absolute min? max?

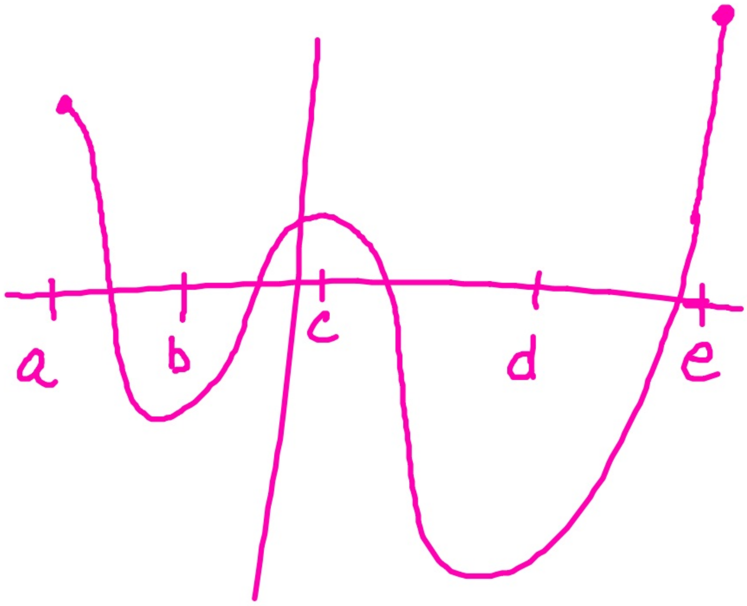


$(-2, 3)$

no abs. max

abs. min at $x = 1$

Relative extrema vs absolute extrema [a, e]



Relative extrema is NEVER
at the endpoints

Abs. max : x = e

Abs. min : x = d

relative max : x = c

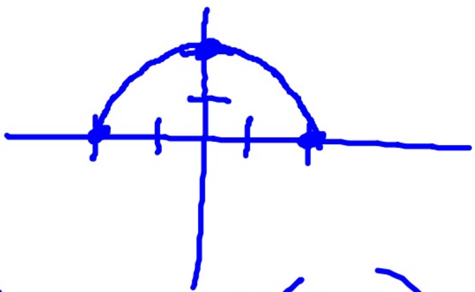
relative min : x = b, d

Absolute extrema can also be a relative extrema

Locate the absolute extrema
(if any) on the stated intervals

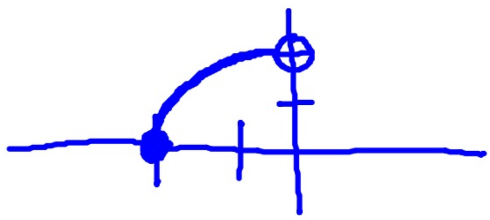
$f(x) = \sqrt{4 - x^2}$

a) [-2, 2]



Abs. max : (0, 2)
Abs. min : (-2,0), (2,0)

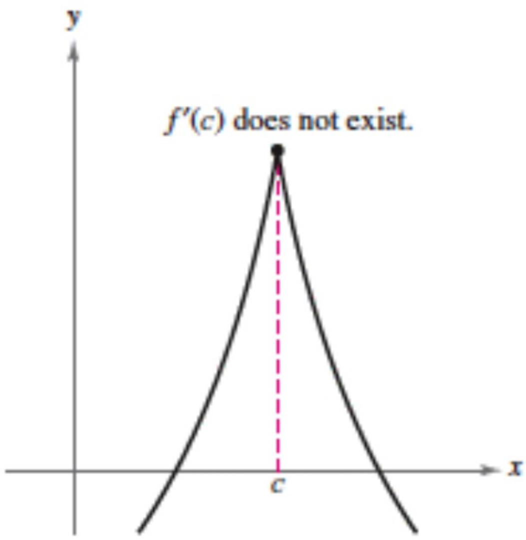
b. [-2, 0)

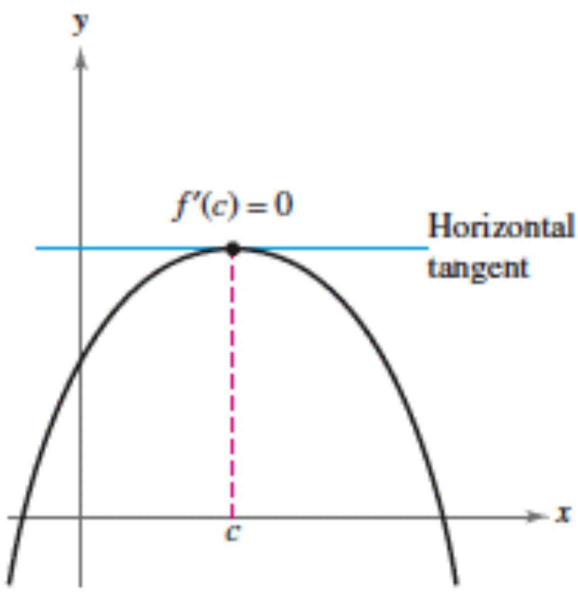


No abs. max.
Abs. min : (-2,0)

DEFINITION OF A CRITICAL NUMBER

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.





#1 Find the critical numbers

$g(x) = x^4 - 4x^2$

$(-\infty, \infty)$

$g'(x) = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

$\boxed{x = 0 \quad x = \pm\sqrt{2}}$

#2 $f(x) = x^3 - 6x^2 + 15$

D: $(-\infty, \infty)$

$f'(x) = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, 4$

#3 $f(x) = 2x - 3x^{2/3}$

D: $(-\infty, \infty)$

$f'(x) = 2 - 2x^{-1/3}$

$= \frac{2}{1} - \frac{2}{x^{1/3}}$

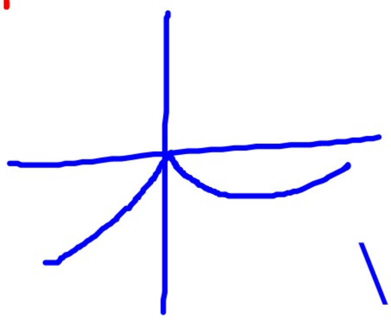
make a common denominator

$f'(x) = \frac{2x^{1/3} - 2}{x^{1/3}}$

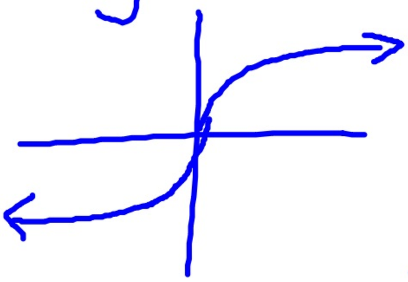
slope is 0

slope is undefined

0 slope	und. slope
$2x^{1/3} - 2 = 0$	$x^{1/3} = 0$
$x^{1/3} = 1$	$x = 0$
$x = 1$	



$y = x^{1/3}$



$y' = \frac{1}{3} x^{-2/3}$

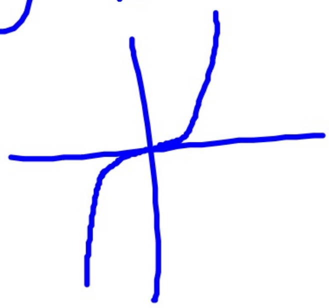
$y' = \frac{1}{3x^{2/3}}$

$y = x^3$

$y' = \frac{3x^2}{1}$

$0 = 3x^2$

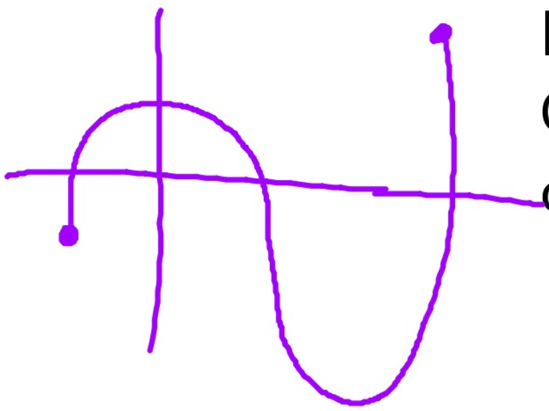
$0 = x$



GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.



Finding absolute extrema: Check endpoints and critical points

#4: Find the absolute extrema.

$$f(x) = \frac{2x}{x^2 + 1} \quad [-2, 2]$$

Check endpoints and critical points

$$f'(x) = \frac{-2(x^2-1)}{(x^2+1)^2}$$

slope 0

$-2(x^2-1) = 0$

$x = \pm 1$

x	$f(x)$	
-2	$-4/5$	
-1	-1	min
1	1	max
2	$4/5$	

Abs. max $(1, 1)$

Abs. min $(-1, -1)$

5.) $f(x) = 2\sin x - \cos 2x \qquad [0, \pi]$

$f'(x) = 2\cos x + 2\sin 2x \qquad \sin 2x = 2\sin x\cos x$

$0 = 2\cos x + 2(2\sin x\cos x)$

$0 = 2\cos x\,(1 + 2\sin x)$

$2\cos x = 0 \qquad\qquad 1 + 2\sin x = 0$

$\cos x = 0 \qquad\qquad \sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}} \qquad\qquad x = \cancel{\frac{7\pi}{6}}, \cancel{\frac{11\pi}{6}}$

	x	$f(x)$
min	0	-1
max	$\frac{\pi}{2}$	$2-(-1)=3$
min	π	-1

Abs max $\left(\frac{\pi}{2}, 3\right)$

Abs. min $(0, -1), (\pi, -1)$

6.) $f(x) = \sqrt{16 - x^2}$ $\quad (-4, 4)$

$f'(x) = \frac{1}{2}(16 - x^2)^{-1/2} \cdot -2x$

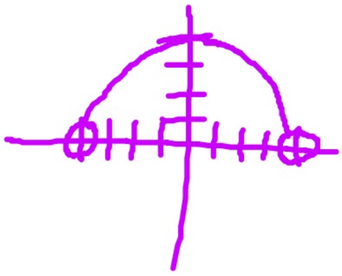
$f'(x) = \dfrac{-x}{\sqrt{16 - x^2}}$

slope zero

$x = 0$

slope und.

$x = \pm 4$



x	y	
-4	0	
0	4	max
4	0	

Abs. max $(0, 4)$

no abs. min.

#5: Find the absolute extrema.

$$g(x) = \sec x, \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$$