2.6 Related Rates

- Find a related rate.
- Use related rates to solve real-life problems.

GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

$$\frac{d}{dt}\left(x^2 + y^2 = 17\right)$$

$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

In Exercises 1–4, assume that x and y are both differentiable functions of t and find the required values of dy/dt and dx/dt.

#1

$x^2 + y^2 = 25$ (a) $\frac{dy}{dt}$ when $x = 3, y = 4$ $\frac{dx}{dt} = 8$

$$\frac{d}{dt}\left(x^2 + y^2 = 25\right)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\cancel{2}(3)(8) + \cancel{2}(4)\frac{dy}{dt} = 0$$

$$24 + 4\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -6$$

#2 ***Volume*** All edges of a cube are expanding at a rate of 6 centimeters per second. How fast is the volume changing when each edge is (a) 2 centimeters ~~and (b) 10 centimeters?~~

rate

$V_{cube} = e^3$

$\frac{de}{dt} = 6$ cm/sec $\quad \frac{dV}{dt} = ?$ when $e = 2$cm

$$\frac{d}{dt}\left(V = e^3\right)$$

$$\frac{dV}{dt} = 3e^2 \cdot \frac{de}{dt}$$

$$= 3(2)^2 \cdot 6 = 72 \text{ cm}^3/\text{sec}$$

$$V_{cube} = e^3$$

$$SA_{cube} = 6e^2$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$SA_{sphere} = 4\pi r^2$$

$$V_{cylinder} = \pi r^2 h$$

$$A_{circle} = \pi r^2$$

$$C = 2\pi r$$

#3 The radius of a circle is increasing at the rate of 5 in/min. At what rate is the area increasing when the radius is 10 in?

$\frac{dr}{dt} = 5$ in/min $\quad \frac{dA}{dt} = ?$ when $r = 10$ in.

$$\frac{d}{dt}\left(A = \pi r^2\right)$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(10)(5)$$

$$= 100\pi \text{ in}^2/\text{min}$$

#4

A spherical balloon is inflated at the rate of 16 cubic feet per min. How fast is the radius changing when the radius is 2 feet?

$$\frac{dV}{dt} = 16\,ft^3/min \qquad \frac{dr}{dt} = \_\_\_ \text{ when } r = 2ft$$

$$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

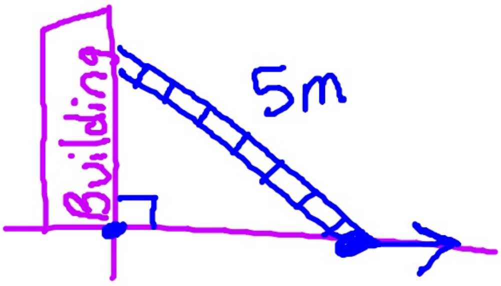
$$16 = 4\pi(2)^2 \frac{dr}{dt}$$

$$16 = 16\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{\pi}$$

ft/min

#5 A 5 meter long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at the rate of 0.4 m/sec. How fast is the top of the ladder descending when the foot of the ladder is 3 m from the house?



$$\frac{dx}{dt} = 0.4 \text{ m/sec}$$

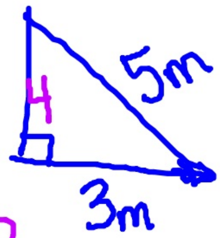
$$\frac{dy}{dt} = \,? \text{ when } x = 3\text{m}$$

$$x^2 + y^2 = c^2$$

$$\frac{d}{dt}\left(x^2 + y^2 = 5^2\right)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

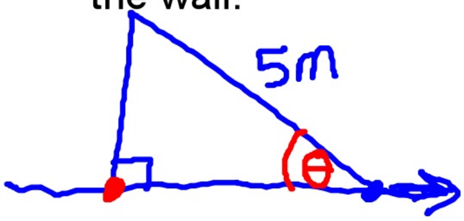


$$3(0.4) + 4\frac{dy}{dt} = 0$$

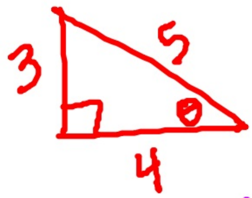
$$4\frac{dy}{dt} = -1.2$$

$$\frac{dy}{dt} = \frac{-1.2}{4} \text{ m/sec}$$

#6 A 5 meter long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at the rate of 0.4 m/sec. Find the rate at which the angle between the ladder and the ground is changing when the base of the ladder is 4 meters from the wall.



$\frac{dx}{dt} = \frac{2}{5}$ m/sec $\quad \frac{d\theta}{dt} = ?$ when $x = 4m$



$$\cos\theta = \frac{x}{h}$$

$$\cos\theta = \frac{x}{5}$$

$$\frac{d}{dt}\left(\cos\theta = \frac{1}{5}x\right)$$

$$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{5} \cdot \frac{dx}{dt}$$

$$\cancel{-\left(\frac{3}{5}\right)} \frac{d\theta}{dt} = \frac{2}{25}\left(-\frac{5}{3}\right)$$

$$\frac{d\theta}{dt} = -\frac{2}{15} \text{ rad/sec}$$

#7

. The radius of a right circular cylinder is increasing at a rate of 2 *in/min* and the height is decreasing at a rate of 3 *in/min.* At what rate is the volume changing when the radius is 8 *in* and the height is 12 *in*? Is the volume increasing or decreasing?

$\frac{dr}{dt} = 2 \text{ in/min}$ $\quad \frac{dh}{dt} = -3 \text{ in/min}$

$\frac{dV}{dt} = \_\_\_$ when $r = 8 \text{in}, h = 12 \text{in}.$

$\frac{d}{dt}\left(V = \pi r^2 h\right)$

$\frac{dV}{dt} = \pi\left(r^2 \cdot \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt}\right)$

$= \pi\left(64 \cdot (-3) + 12 \cdot 16 \cdot 2\right)$

$= \pi\left(-192 + 384\right) = 192\pi \text{ in}^3/\text{min}$