2.5 Implicit Differentiation

- Distinguish between functions written in implicit form and explicit form.
- Use implicit differentiation to find the derivative of a function.

Implicit Form	*Explicit Form*	*Derivative*
$xy = 1$	$y = \frac{1}{x} = x^{-1}$	$\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$

Up to now, we hae been finding derivatives of functions explicitly. You can also find derivatives of equations that are not functions implicitly. Or, if you wish, you can find derivatives of functions implicitly too :)

Find the equation of the tangent line at the given point.

HW: 21, 22, 29, 45, 47

#2 $\frac{d}{dx}\left(x^3 + y^3 = 6xy - 1\right) \quad (2, 3)$

$3x^2\frac{dx}{dx} + 3y^2\frac{dy}{dx} = 6\left(x\frac{dy}{dx} + y\cdot\frac{dx}{dx}\right)$

$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$

$x^2 + y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y$

$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$

$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{6-4}{9-4} = \frac{2}{5}$

$y - 3 = \frac{2}{5}(x - 2)$

Find the slope at the given point.

#1 $\frac{d}{dx}(x\cos y = 1) \quad \left(2, \frac{\pi}{3}\right)$

$$x(-\sin y)\frac{dy}{dx} + \cos y \cdot 1\frac{dx}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\cos y}{x\sin y} = \frac{\cot y}{x}$$

$$\left.\frac{dy}{dx}\right|_{\left(2,\frac{\pi}{3}\right)} = \frac{\cot\frac{\pi}{3}}{2} = \frac{\frac{\sqrt{3}}{3}}{2} = \frac{\sqrt{3}}{6}$$

#3: Given $x^2 + y^2 = 4$, show that $\frac{d^2y}{dx^2} = \frac{-4}{y^3}$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{-x}{y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\cdot 1\frac{dy}{dx}}{y^2}$$

$$= \left(\frac{-y + x\left(\frac{-x}{y}\right)}{y^2}\right)\frac{y^2}{y^2}$$

$$\frac{-y^3 - x^2y}{y^4}$$

$$\frac{-y(y^2 + x^2)}{y^4}$$

$$\frac{-4y}{y^4} = \frac{-4}{y^3}$$

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#4: Find all points where the graph has vertical and horizontal tangents. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$$x^3y^3 - y = x$$