2.4 The Chain Rule

- Find the derivative of a composite function using the Chain Rule.
- Find the derivative of a function using the General Power Rule.
- Simplify the derivative of a function using algebra.
- Find the derivative of a trigonometric function using the Chain Rule.

The Chain Rule is for taking derivatives of composite functions such as:

$h(x) = (3x - 1)^6$ and $s(x) = \sin(5x)$

These functions have an outer and inner function.

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Find dy/dx

#1: $y = (2x - 3)^2$

$y = 4x^2 - 12x + 9$

$y' = 8x - 12$

$y' = 4(2x-3)$

$y = (2x-3)^2$

$y' = 2(2x-3)^1 \cdot 2$

$y' = 4(2x-3)$

Differentiate.

#2: $y = (2x - 3)^5$

$$y' = 5(2x-3)^4 \cdot 2$$

$$y' = 10(2x-3)^4$$

#3 Find the derivative. Then find y'(2)

$$y = \frac{4}{\sqrt{x^2+1}}$$

$$y = 4(x^2+1)^{-1/2}$$

$$y' = -2(x^2+1)^{-3/2} \cdot 2x$$

$$y'(2) = \frac{-8}{5^{3/2}} = \frac{-8}{\sqrt{125}}$$

$$= -8 \cdot 5^{-3/2}$$

#4 $s(t) = \frac{1}{t^2 + 3t - 1}$

$s(t) = (t^2 + 3t - 1)^{-1}$

$s'(t) = -1(t^2 + 3t - 1)^{-2} \cdot (2t+3)$

$s'(t) = \frac{-(2t+3)}{(t^2+3t-1)^2}$

#5 $y = \sin(\pi x)$

$y' = \cos(\pi x) \cdot \pi$

$y' = \pi \cos(\pi x)$

#6: $y = \sec(x^2)$

$y' = \sec(x^2)\tan(x^2) \cdot 2x$

$y' = 2x\sec(x^2)\tan(x^2)$

#7: $y = \cos^3 x$

$$y = (\cos x)^3$$

$$y' = 3(\cos x)^2(-\sin x)$$

$$y' = -3\cos^2 x \sin x$$