Find $f'(x)$

1. $f(x) = \frac{x^2}{x^2+1}$

2. $f(x) = \sin x \tan x$

3. $f(x) = \frac{\cos x}{1+\sin x}$

4. $f(x) = \cot x - \csc x$

5. $f(x) = \frac{\cos x}{x^3}$

6. $f(x) = x^2 \sec x$

(1) $f'(x) = \dfrac{(x^2+1)2x - x^2(2x)}{(x^2+1)^2}$

$f'(x) = \dfrac{2x}{(x^2+1)^2}$

(2) $f'(x) = \sin x \sec^2 x + \tan x \cos x$

(3) $f'(x) = \dfrac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2} = \dfrac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$

$= \dfrac{-\sin x - 1}{(1+\sin x)^2}$

(4) $f'(x) = -\csc^2 x + \csc x \cot x$

(5) $f'(x) = \dfrac{x^3(-\sin x) - \cos x(3x^2)}{x^6}$

$= \dfrac{-x^2(x\sin x + 3\cos x)}{x^6}$

$= -\dfrac{(x\sin x + 3\cos x)}{x^4}$

(5) $f(x) = x^{-3}\cos x$

product rule $f'(x) = x^{-3}(-\sin x) + \cos x(-3x^{-4})$

(6) $f'(x) = x^2 \sec x \tan x + \sec x \cdot 2x$

$= x \sec x (x + \tan x + 2)$

(7) Write an equation of the tangent line to $y = 2\cot x$ at $\left(\frac{\pi}{3}, \frac{2\sqrt{3}}{3}\right)$

(8) find $f''(4)$

$f(x) = 3\sqrt{x} - \frac{1}{3}x^3$

(7) Write an equation of the tangent line to $y = 2\cot x$ at $\left(\frac{\pi}{3}, \frac{2\sqrt{3}}{3}\right)$

$y' = -2\csc^2 x$

$y'\left(\frac{\pi}{3}\right) = -2\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{-8}{3}$

$$\boxed{y - \frac{2\sqrt{3}}{3} = \frac{-8}{3}\left(x - \frac{\pi}{3}\right)}$$

(8) find $f''(4)$

$f(x) = 3\sqrt{x} - \frac{1}{3}x^3$

$f(x) = 3x^{1/2} - \frac{1}{3}x^3$

$f'(x) = \frac{3}{2}x^{-1/2} - x^2$

$f''(x) = -\frac{3}{4}x^{-3/2} - 2x$

$f''(4) = \frac{-3}{4(4)^{3/2}} - 2(4)$

$= -\frac{3}{32} - 8 = -8\frac{3}{32}$

$-\frac{3}{32} - \frac{256}{32} = -\frac{259}{32}$ or

(9) $g(5) = -3$

$g'(5) = 6$

$h(5) = 3$

$h'(5) = -2$

$m(x) = g(x)h(x)$

$m'(5) =$

$r(x) = \frac{g(x)}{h(x)}$

$r'(5) =$

(9) $g(5) = -3$

$g'(5) = 6$

$h(5) = 3$

$h'(5) = -2$

$m(x) = g(x)h(x)$

$m'(5) = g(5)h'(5) + h(5)g'(5)$

$= (-3)(-2) + (3)(6)$

$= 24$

$r(x) = \frac{g(x)}{h(x)}$

$r'(5) = \frac{h(5)g'(5) - g(5)h'(5)}{(h(5))^2}$

$= \frac{3(6) - (-3)(-2)}{9} = 4/3$