

$$76.) f'(x) = \frac{(x^2-7)(1) - (x-4)(2x)}{(x^2-7)^2}$$

$$= \frac{x^2 - 7 - 2x^2 + 8x}{(x^2-7)^2}$$

$$0 = \frac{-x^2 + 8x - 7}{(x^2-7)^2} \quad \left(1, \frac{-3}{-6}\right)$$
$$\quad \quad \quad \left(7, \frac{3}{42}\right)$$

$$0 = \frac{x^2 - 8x + 7}{(x-7)(x-1)}$$

$$37.) f'(x) = \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2}$$

$$= \frac{\cancel{2x^3} - 2c^2x - \cancel{2x^3} - 2c^2x}{(x^2 - c^2)^2}$$

$$= \frac{-4c^2x}{(x^2 - c^2)^2}$$

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$$29.) f(x) = \frac{3x^1 - 1}{\sqrt{x}}$$

$$f(x) = 3x^{1/2} - x^{-1/2}$$

$$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

2.3: Product/Quotient/Trig Rules

1) Find the derivative of $y = \tan x$

$$y = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$y' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$y' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$y' = \frac{1}{\cos^2 x} = \sec^2 x$$

#2: Find the derivative of $y = \sec x$

$$y = \frac{1}{\cos x}$$

$$y' = \frac{\cos x(0) - (1)(-\sin x)}{\cos^2 x}$$

$$y' = \frac{\sin x}{\cos^2 x} = \frac{\sin x \cdot 1}{\cos x \cdot \cos x}$$

$$y' = \tan x \sec x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

#3 (Yes, we did this problem yesterday)

$$y = \frac{1 - \cos x}{\sin x} = \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \csc x - \cot x$$

$$y' = -\csc x \cot x + \csc^2 x$$

$\tan x \sec x \sec x$ $- \cot x \csc x \csc x$

#4: Find the equation of the tangent line to the point.

$$h(t) = \frac{\sec t}{t}$$

$$\left(\pi, -\frac{1}{\pi}\right)$$

$$h'(t) = \frac{t \cdot \tan t \sec t - \sec t \cdot 1}{t^2}$$

$$h'(\pi) = \frac{\pi \tan \pi \sec \pi - \sec \pi}{\pi^2}$$

$$= \frac{0 - (-1)}{\pi^2} = \frac{1}{\pi^2}$$

$$y + \frac{1}{\pi} = \frac{1}{\pi^2}(x - \pi)$$

Higher Order Derivatives

$$\#5 \quad f(x) = x^4 - 2x^3 + 7x + 9$$

$$f'(x) = 4x^3 - 6x^2 + 7$$

$$f''(x) = 12x^2 - 12x$$

$$f'''(x) = 24x - 12$$

$$f^{(4)}(x) = 24$$

$$f^{(5)}(x) = 0$$

#6: Find $f'(x)$

$$f(x) = \sec x$$

$$f'(x) = \tan x \sec x$$

$$\begin{aligned} f''(x) &= \tan x \cdot \tan x \sec x + \sec x \cdot \sec^2 x \\ &= \tan^2 x \sec x + \sec^3 x \end{aligned}$$

#7: Find $f'(2)$

$$g(2) = 3 \quad \text{and} \quad g'(2) = -2 \quad \cdot \quad f(x) = g(x)h(x)$$

$$h(2) = -1 \quad \text{and} \quad h'(2) = 4$$

$$f(x) = g(x)h(x)$$

$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

$$f'(2) = g(2)h'(2) + h(2)g'(2)$$

$$= (3)(4) + (-1)(-2)$$

$$= 14$$