2.3 Product and Quotient Rules and Higher-Order Derivatives

- Find the derivative of a function using the Product Rule.
- Find the derivative of a function using the Quotient Rule.
- Find the derivative of a trigonometric function.
- Find a higher-order derivative of a function.

The Product Rule

$$\frac{d}{dx}[fg] = f \cdot g' + g \cdot f'$$

$y = \overset{f}{x^2}\overset{g}{\sin x}$

$y' = x^2 \cdot \cos x + \sin x \cdot 2x$

$y' = x^2 \cos x + 2x \sin x$

#1

$$g(x) = (3x - 1)(4x + 5)$$

$g(x) = 12x^2 + 11x - 5$

$g'(x) = 24x + 11$

$g'(x) = (3x-1)4 + (4x+5)\cdot 3$

$g'(x) = 12x - 4 + 12x + 15$

$g'(x) = 24x + 11$

$y = 5\sin x$

$y' = 5 \cdot \cos x$

$y' = \cancel{5 \cdot \cos x + \sin x \cdot 0}$

#2: Write the equation of the line tangent to f(x) at x = 0

$f(x) = \sin^{f} x \cos^{g} x$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x)$

$f'(x) = -\sin^2 x + \cos^2 x$

$f'(0) = 0 + 1$

$f'(0) = 1$

$(0, 0)\ m = 1$

$y - 0 = 1(x - 0)$

$y = x$

Find f'(x) and f'(c)

#3 $f(x) = x\cos x$ $\qquad c = \frac{\pi}{4}$

$f'(x) = x(-\sin x) + \cos x(1)$

$f'\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}$

$f'\left(\frac{\pi}{4}\right) = \frac{-\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$

#4 $f(x) = \frac{\sin x}{x} = \sin x \cdot x^{-1}$ $\qquad c = \frac{\pi}{6}$

$f'(x) = \sin x \cdot -1x^{-2} + x^{-1} \cdot \cos x$

$f'(x) = -\frac{\sin x}{x^2} + \frac{\cos x}{x}$ $\qquad f'\left(\frac{\pi}{6}\right) = \frac{-\frac{1}{2}}{\left(\frac{\pi}{6}\right)^2} + \frac{\frac{\sqrt{3}}{2}}{\frac{\pi}{6}}$

$$f'\left(\frac{\pi}{6}\right) = \frac{-\frac{1}{2}}{\left(\frac{\pi}{6}\right)^2} + \frac{\frac{\sqrt{3}}{2}}{\frac{\pi}{6}} = \frac{-\frac{1}{2}}{\frac{\pi^2}{36}} + \frac{\frac{\sqrt{3}}{2}}{\frac{\pi}{6}}$$

$$= \frac{-1}{2} \cdot \frac{36}{\pi^2} + \frac{\sqrt{3}}{2} \cdot \frac{6}{\pi}$$

$$f'\left(\frac{\pi}{6}\right) = \frac{-18}{\pi^2} + \frac{3\sqrt{3}}{\pi}$$

5.) $y = \sqrt{x}(x^2 - 7)$

$y = x^{5/2} - 7x^{1/2}$

$y' = \frac{5}{2}x^{3/2} - \frac{7}{2}x^{-1/2}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-1	1	2
2	1	$\frac{1}{2}$	3	0
3	3	2	1	-2

$h'(1)$

Given $h(x) = f(x) - g(x)$, find $h'(2)$

$h'(x) = f'(x) - g'(x)$

$h'(2) = f'(2) - g'(2)$

$= \frac{1}{2} - 0$

$= \frac{1}{2}$

Given $h(x) = f(x) \cdot g(x)$, find $h'(3)$

$h'(x) = f(x)\,g'(x) + g(x)f'(x)$

$h'(3) = f(3)g'(3) + g(3)f'(3)$

$= (3)(-2) + (1)(2)$

$= -4$