2.3 Product and Quotient Rules and Higher-Order Derivatives

- Find the derivative of a function using the Product Rule.
- Find the derivative of a function using the Quotient Rule.
- Find the derivative of a trigonometric function.
- Find a higher-order derivative of a function.

The Product Rule

$$\frac{d}{dx}[f \cdot g] = f \cdot g' + g \cdot f'$$

HW Pg. 126: 1, 3, 17, 31, 53, 63a

#1

$$f(x) = (6x + 5)(x^3 - 2)$$

$f(x) = 6x^4 + 5x^3 - 12x - 10$

$f'(x) = 24x^3 + 15x^2 - 12$

$= 3(8x^3 + 5x^2 - 4)$

did not use product rule

#1

$$f(x) = \overset{f}{(6x+5)}\,\overset{g}{(x^3-2)}$$

$$f'(x) = (6x+5)3x^2 + (x^3-2)6$$

$$= 18x^3 + 15x^2 + 6x^3 - 12$$

$$= 24x^3 + 15x^2 - 12$$

#2 $f(x) = \overset{f}{\sin x}\,\overset{g}{\cos x}$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x)$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

optional

Do you need to use product rule to find the derivative?

#3

$$f(x) = (x^2 - 3)^2$$

No

$f(x) = x^4 - 6x^2 + 9$

$f'(x) = 4x^3 - 12x$

#4

$$f(x) = 4x^3$$

No

$f'(x) = 12x^2$

$f(x) = 4x^3$

$f'(x) = 4(3x^2) + \cancel{x^3}(0)$

#5

$$f(x) = x\sin x$$

$$f'(x) = x\cos x + \sin x \cdot 1$$

#6

$$f(x) = \frac{\sqrt{x} - 1}{x}$$

$$\cancel{f(x) = x^{-1}\left(\sqrt{x} - 1\right)}$$

$$f(x) = x^{-1/2} - x^{-1}$$

$$f'(x) = -\frac{1}{2}x^{-3/2} + 1x^{-2}$$

#7

$$f(x) = \sqrt{x}(x-2)$$

$$f(x) = x^{3/2} - 2x^{1/2}$$

$$f'(x) = \frac{3}{2}x^{1/2} - 1x^{-1/2}$$

Find the equation of the tangent line to f(x) at the indicated x-value

#8 $f(x) = \dfrac{\sin x}{x}; x = \dfrac{\pi}{2}$

$\left(\frac{\pi}{2}, \frac{2}{\pi}\right)$

$f(x) = \sin x \cdot x^{-1}$

$f'(x) = \sin x\left(-x^{-2}\right) + x^{-1}\cos x$

$f'\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2}\left(\dfrac{-1}{\left(\frac{\pi}{2}\right)^2}\right) + 0$

$= (1)\left(\dfrac{-4}{\pi^2}\right)$

$\boxed{y - \frac{2}{\pi} = \frac{-4}{\pi^2}\left(x - \frac{\pi}{2}\right)}$