2.2 Basic Differentiation Rules and Rates of Change

- Find the derivative of a function using the Constant Rule.
- Find the derivative of a function using the Power Rule.
- Find the derivative of a function using the Constant Multiple Rule.
- Find the derivative of a function using the Sum and Difference Rules.
- Find the derivatives of the sine function and of the cosine function.
- Use derivatives to find rates of change.

Find the slope

Find the equation of a tangent line

Find where a function has a horizontal slope

Differentiability

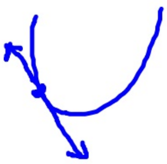
#1 Find the slope at the indicated point.

$f(x) = x^2 + 3x + 4, \quad (-2, 2)$

$f'(x) = 2x + 3$

$f'(-2) = -1$

$\cancel{m = -1}$



$32^{3/5}$

$(2^{\cancel{5}})^{\cancel{3/5}}$

8

$f(x) = \dfrac{4}{\sqrt{x}} \quad (4,2)$

$f(x) = 4x^{-1/2}$

$f'(x) = -2x^{-3/2}$

$f'(4) = \dfrac{-2}{4^{3/2}}$

$= \dfrac{-2}{8} = \dfrac{-1}{4}$

#2 Find the equation of the tangent line at the point indicated.

$f(x) = x^2 + 3x + 4, \quad (-2, 2)$

$f'(-2) = -1$

Point-slope form

$y - y_1 = m(x - x_1)$

$\boxed{y - 2 = -1(x + 2)}$

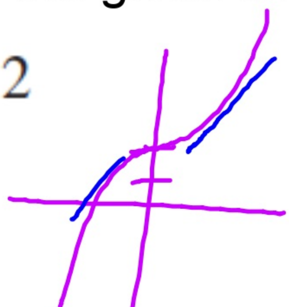
$f(x) = \frac{4}{\sqrt{x}} \quad (4,2)$

$f'(4) = \frac{-1}{4}$

$\boxed{y - 2 = \frac{-1}{4}(x - 4)}$

\#3 Find the equation of the line tangent to the graph of f and parallel to the given line.

$f(x) = x^3 + 2$



$3x - y - 4 = 0$

$y = 3x - 4$

$y' = 3$

$f'(x) = 3x^2$

$3 = 3x^2$

slope of line

slope of f(x)

$1 = x^2$

$\pm 1 = x$

There will be two points that have a slope of 3.

$(1, 3)$ $(-1, 1)$

$\boxed{y - 3 = 3(x - 1)}$ $\boxed{y - 1 = 3(x + 1)}$

#4

Determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

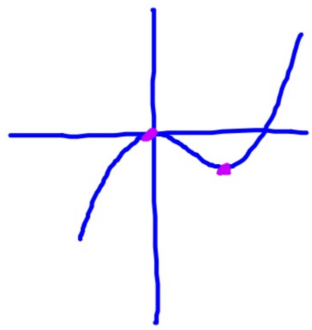
$f(x) = x^3 - \frac{3}{2}x^2$ **where the function has slope 0.**

$f'(x) = 3x^2 - 3x^1$

$0 = 3x(x-1)$

$x = 0, 1$

$$\boxed{\begin{array}{c}(0, 0) \\ (1, -\frac{1}{2})\end{array}}$$



#5: Determine if f(x) is differentiable at x = 3.

$$f(x) = \begin{cases} -x+4 & x<3 \\ x^2-8 & x \geq 3 \end{cases}$$

$$f'(x) = \begin{cases} -1, & x<3 \\ 2x, & x \geq 3 \end{cases}$$

Check continuity first

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$$

$$1 = 1$$

Check differentiability

$$\lim_{x\to 3^-} f'(x) = \lim_{x\to 3^+} f'(x) = f'(3)$$

$$-1 \neq 6$$

No, f(x) is not diff. at x = 3

#6

Find a and b so that f(x) is differentiable.

$$f(x) = \begin{cases} ax^2 - 3x & x < 1 \\ bx - 2 & x \geq 1 \end{cases}$$

$$f'(x) = \begin{cases} 2ax - 3, & x < 1 \\ b, & x \geq 1 \end{cases}$$

Continuity

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$a - 3 = b - 2$$

Diff.

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x) = f'(1)$$

$$2a - 3 = b$$

$$\boxed{\begin{array}{l} a = 2 \\ b = 1 \end{array}}$$

33.) $f(x) = -\frac{1}{2} + \frac{7}{5}x^3$ $\quad (0, -\frac{1}{2}$

$f'(x) = 0 + \frac{21}{5}x^2$

$f'(0) = 0$

23.) $y = \frac{1}{x} - 3\sin x$

$y = x^{-1} - 3\sin x$

$y' = -1x^{-2} - 3\cos x$

$= \frac{-1}{x^2} - 3\cos x$

31.) $f(x) = \dfrac{8}{x^2}$ $(2, 2)$

$$f(x) = 8x^{-2}$$

$$f'(x) = -16x^{-3}$$

$$f'(x) = \frac{-16}{x^3}$$

$$f'(2) = \frac{-16}{8}$$

$$= -2$$

27.) $y = \frac{6}{(5x)^3}$

$y = \frac{6}{125x^3}$

$y = \frac{6}{125}x^{-3}$

$y' = \frac{-18}{125}x^{-4}$

39.) $f(x) = x^2 + 5 - 3x^{-2}$

$f'(x) = 2x + 0 + 6x^{-3}$