2.1 The Derivative and the Tangent Line Problem

- Find the slope of the tangent line to a curve at a point.
- Use the limit definition to find the derivative of a function.
- Understand the relationship between differentiability and continuity.

Alternate Form of the Derivative

(provided the limit exists)

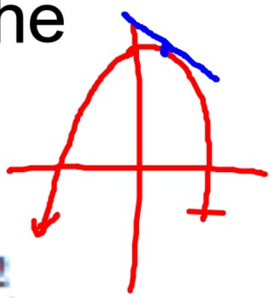
$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Note: This is a general limit!!!

f'(c): the slope at a given point on f(x)

The answer will be numerical (provided the limit exists)

Find g'(1) using the alternate form of the derivative



Hint: we did this problem earlier

$g'(1) = -2$

#1 $g(x) = 6 - x^2$

$g'(x) = -2x$

$g'(1) = -2$

$$g'(c) = \lim_{x \to c} \frac{g(x) - g(c)}{x - c}$$

$$g'(1) = \lim_{x \to 1} \frac{g(x) - g(1)}{x - 1} = \lim_{x \to 1} \frac{6 - x^2 - 5}{x - 1}$$

$$= \lim_{x \to 1} \frac{1 - x^2}{x - 1} = \lim_{x \to 1} \frac{-(x^2 - 1)}{x - 1} = \lim_{x \to 1} \frac{-\cancel{(x-1)}(x+1)}{\cancel{x-1}} = -2$$

$f(-1) = -1$

$f(x) = 2x^2 + 3x$

$f'(-1) = \_\_\_\_$

$$f'(-1) = \lim_{x \to -1} \frac{2x^2 + 3x + 1}{x + 1}$$

$$= \lim_{x \to -1} \frac{(2x+1)\cancel{(x+1)}}{\cancel{x+1}}$$

$$= -1$$

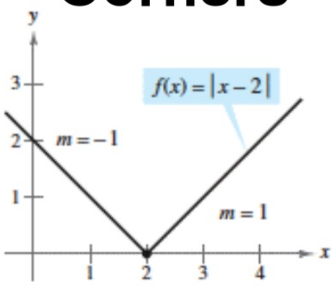
$$\boxed{f'(-1) = -1}$$

Differentiability

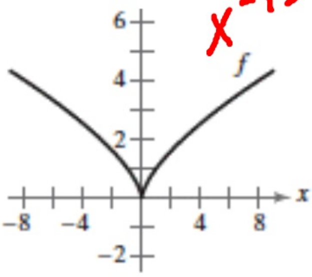
A function is considered differentiable at a point if the derivative can be found.

A function is not differentiable when the function has:

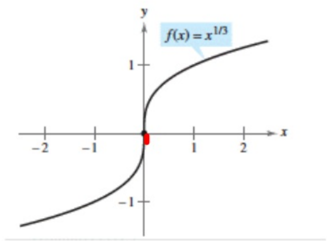
Corners



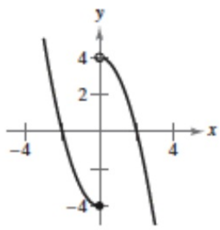
Cusps $x^{2/3}$

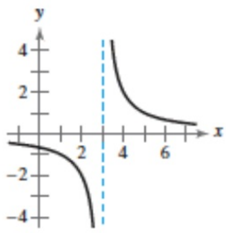


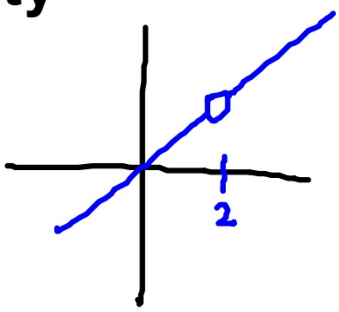
Vertical Tangents



any type of discontinuity





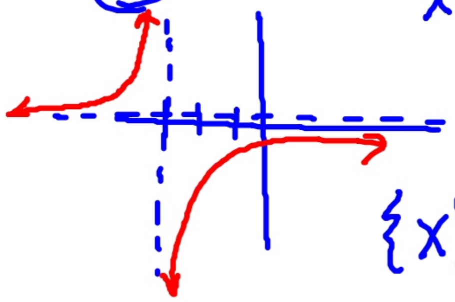


Describe x-values at which f(x) is differentiable.

1. $f(x) = |x-2| - 3$

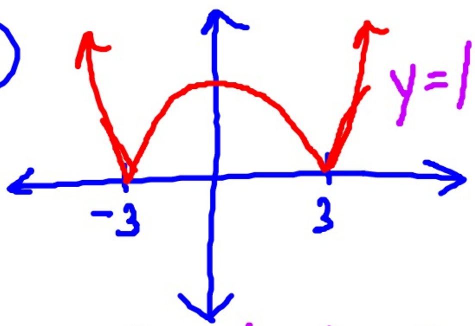
$(-\infty, 2) \cup (2, \infty)$

2. $f(x) = \frac{-1}{x+3}$



$\{x \mid x \neq -3\}$

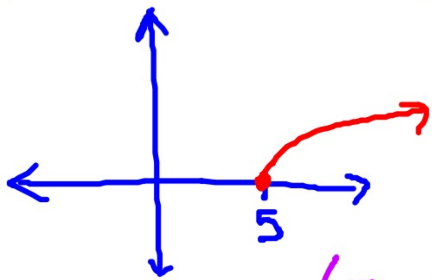
3. $y = |x^2 - 9|$



$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

$\{x \mid x \neq -3, 3\}$

4. $f(x) = \sqrt{x-5}$

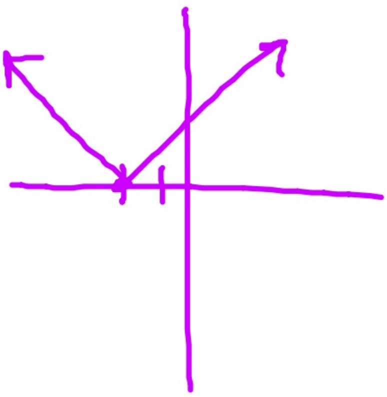


$(5, \infty)$

$\{x \mid x > 5\}$

Use the alternate form of the derivative to find the derivative at x = c (if it exists).

$g(x) = |x+2| \quad c = -2$



$$g'(-2) = \lim_{x \to -2} \frac{|x+2|}{x+2}$$

dne

not differentiable at $x = -2$

True or false.
Explain.

If $f(x)$ is continuous at $x = c$,then $f(x)$ is differentiable at $x = c$

If $f(x)$ is differentiable at $x = c$, then $f(x)$ is continuous at $x = c$.

Differentiability implies continuity BUT continuity does not imply differentiability!!!!!!!!!!!