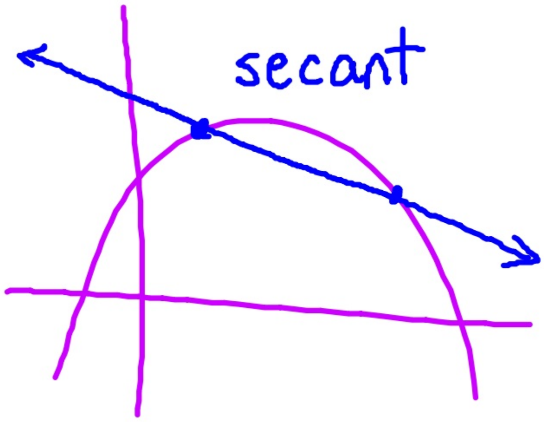
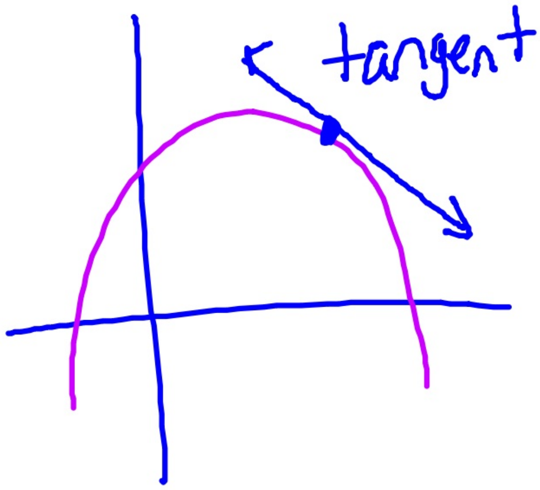
2.1 The Derivative and the Tangent Line Problem

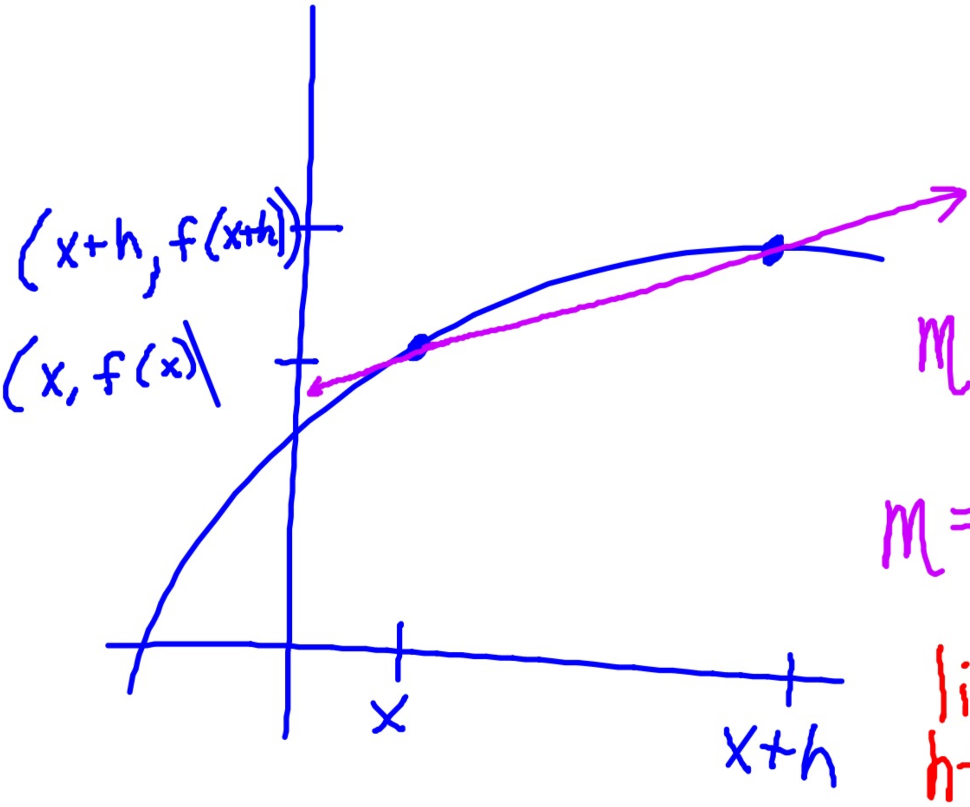
- Find the slope of the tangent line to a curve at a point.
- Use the limit definition to find the derivative of a function.
- Understand the relationship between differentiability and continuity.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

slope between 2 points







$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{f(x+h) - f(x)}{x+h-x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

h : very small number

DEFINITION OF THE DERIVATIVE OF A FUNCTION

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

$f'(x)$
derivative
of
$f(x)$

p. 104
13-23 odd
skip 19

Find the derivative by the limit process.

#1 $f(x) = 3x + 2$

$f(x+h) = 3(x+h) + 2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h) + 2 - (3x+2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x} + 3h + \cancel{2} - \cancel{3x} - \cancel{2}}{h} = \lim_{h \to 0} 3 = 3$$

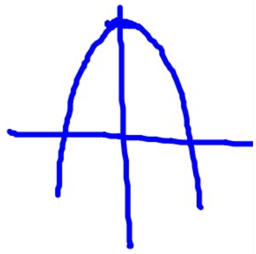
$$\boxed{f'(x) = 3}$$

Ways to express derivatives

$$f'(x),\quad \frac{dy}{dx},\quad y',\quad \frac{d}{dx}[f(x)],\quad D_x[y].$$

$$\begin{aligned}\frac{dy}{dx} &= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}\\ &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}\\ &= f'(x).\end{aligned}$$

#2 $g(x) = 6 - x^2$



$g(x+h) = 6 - (x+h)^2$

$$g'(x) = \lim_{h\to 0} \frac{6-(x+h)^2 - (6-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{6} - (\cancel{x^2} + 2xh + h^2) - \cancel{6} + \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{-2xh - h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = -2x$$

$g'(x) = -2x$

#3 $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

#4

$$f(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h}\,x(x+h) - \frac{1}{x}\,x(x+h)}{h\,x(x+h)}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{h\,x(x+h)}$$

$$= \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - h}{h\,x(x+h)} = \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= -\frac{1}{x^2}$$

#5 $g(x) = 2x^2 - 3x + 1$

$$g'(x) = \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$$

$$g'(x) = 4x - 3$$

Synonyms for derivative

Slope
Rate of Change
Velocity
Rise/Run