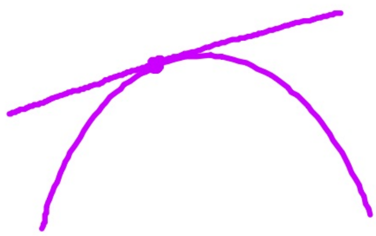
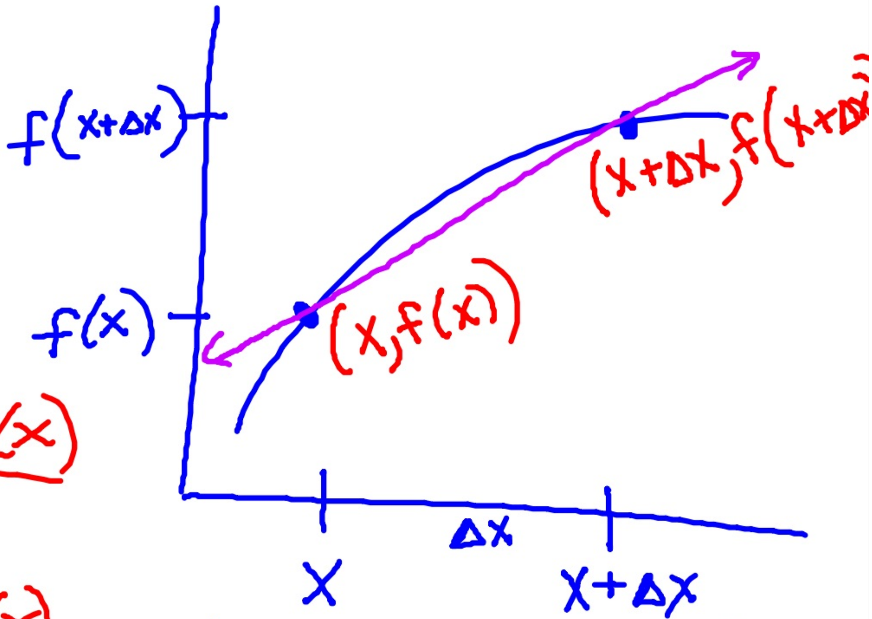
2.1 The Derivative and the Tangent Line Problem

- Find the slope of the tangent line to a curve at a point.
- Use the limit definition to find the derivative of a function.
- Understand the relationship between differentiability and continuity.

tangent line



secant line



$$\text{Slope} = \frac{f(x+\Delta x) - f(x)}{x+\Delta x - x}$$

$$m_{secant} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(difference quotient)

$\Delta x \to 0$

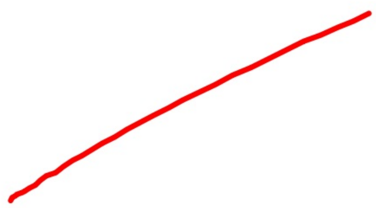
$\Delta x = .00001$

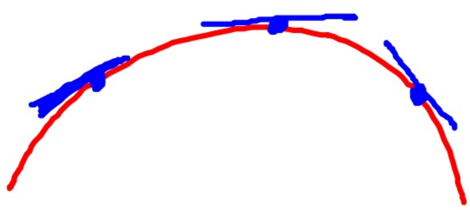
ION OF THE DERIVATIVE OF A FUNCTION

vative of f at x is given by

$$= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

the limit exists. For all x for which this limit exists, f' is a fu





Find the derivative by the limit process
(by definition of the derivative)

$f(x+\Delta x) = 3(x+\Delta x) + 2$

#1 $f(x) = 3x + 2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3(x+\Delta x) + 2 - (3x+2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3x} + 3\Delta x + \cancel{2} - \cancel{3x} - \cancel{2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 3 = 3$$

$f'(x) = 3$

Ways to express derivatives

$$f'(x), \quad \frac{dy}{dx}, \quad y', \quad \frac{d}{dx}[f(x)], \quad D_x[y].$$

$$\begin{aligned}\frac{dy}{dx} &= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= f'(x).\end{aligned}$$

#2 $g(x) = 6 - x^2$ $g(x+h) = 6-(x+h)^2$

$$g'(x) = \lim_{h\to 0} \frac{6-(x+h)^2-(6-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{6-(x^2+2xh+h^2)-6+x^2}{h}$$

$$= \lim_{h\to 0} \frac{-2xh-h^2}{h}$$

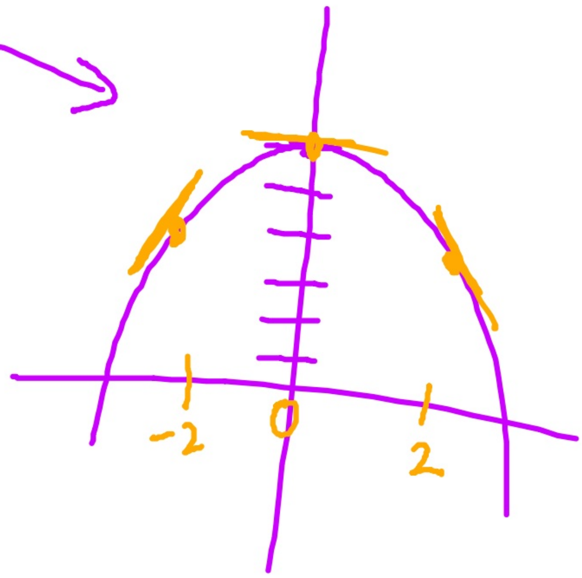
$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = -2x$$

$$\boxed{g'(x) = -2x}$$

$$f(x) = 6 - x^2$$

$$f'(x) = -2x$$

"slope generator"



$$f'(-2) = 4$$

$$f'(0) = 0$$

$$f'(2) = -4$$

#3 $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 5x - 2x^2 \qquad f(x+h) = 5(x+h) - 2(x+h)^2$$

$$f'(x) = \lim_{h \to 0} \frac{5(x+h) - 2(x+h)^2 - (5x - 2x^2)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{5x} + 5h - \cancel{2x^2} - 4xh - 2h^2 - \cancel{5x} + \cancel{2x^2}}{h}$$

$$= \lim_{h \to 0} (5 - 4x - 2h) = 5 - 4x$$

$$f'(x) = 5 - 4x$$

#4 $f(x) = \sqrt{x}$, $f(x+h) = \sqrt{x+h}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

#5 $g(x) = \frac{1}{x}$ $g(x+h) = \frac{1}{x+h}$

$$g'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \lim_{h \to 0} \frac{x-(x+h)}{xh(x+h)} = \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - \cancel{h}}{x\cancel{h}(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

#6 $f(x)=x^3$ $\quad f(x+h)=(x+h)^3$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$=\lim_{h\to 0}(3x^2+3xh+h^2)=3x^2$$

Synonyms for derivative

Slope
Rate of Change
Velocity
Rise/Run