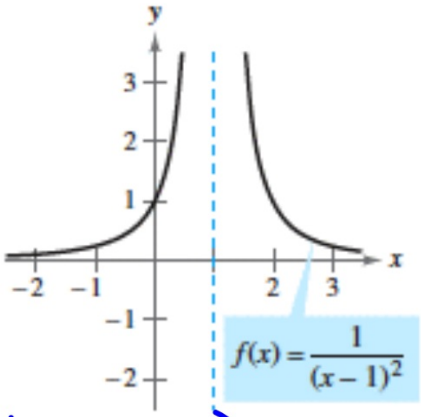
1.5 Infinite Limits

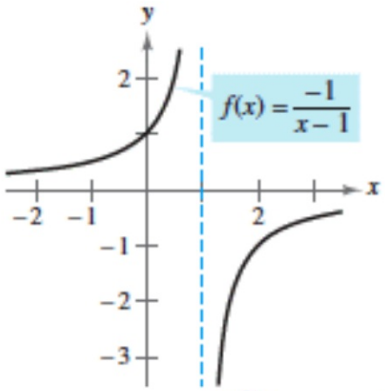
- Determine infinite limits from the left and from the right.
- Find and sketch the vertical asymptotes of the graph of a function.

Determine whether f(x) approaches infinity or negative infinity from the left and the right of the vertical asymptotes.



$\lim_{x \to 1^-} f(x) \quad \infty$

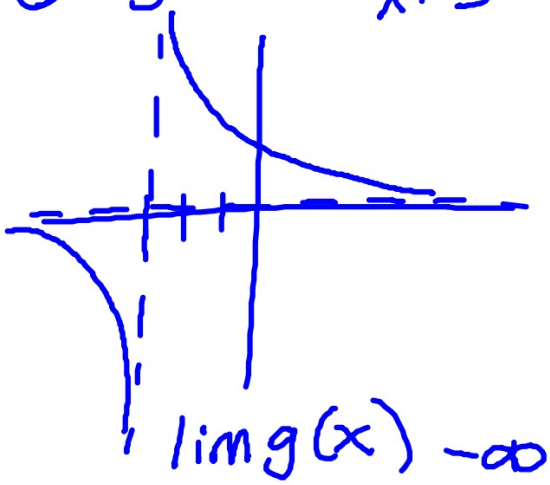
$\lim_{x \to 1^+} f(x) \quad \infty$



$\lim_{x \to 1^-} f(x) \quad \infty$

$\lim_{x \to 1^+} f(x) \quad -\infty$

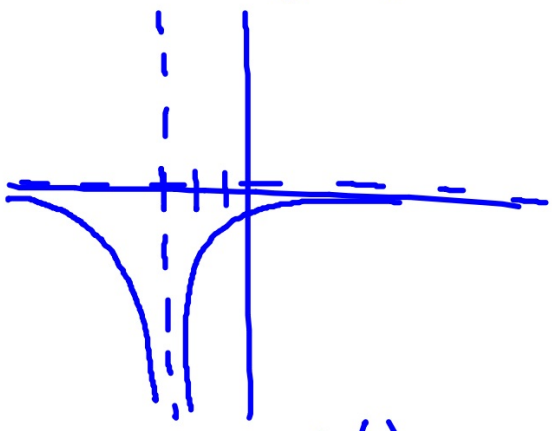
① $g(x) = \frac{1}{x+3}$



$\lim_{x \to -3^-} g(x) = -\infty$

$\lim_{x \to -3^+} g(x) = \infty$

② $h(x) = \frac{-1}{(x+3)^2}$



$\lim_{x \to -3^-} h(x) = -\infty$

$\lim_{x \to -3^+} h(x) = -\infty$

If a function is approaching a vertical asymptote, your answer will be one of these:

∞

$-\infty$

Does not exist

DEFINITION OF VERTICAL ASYMPTOTE

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f.

Look in the denominator for VA.

Determine all vertical asymptotes of the graph of each function.

a. $f(x) = \dfrac{1}{2(x+1)}$

$x = -1$

b. $f(x) = \dfrac{x^2+1}{x^2-1}$

$(x+1)(x-1)$

$x = -1$

$x = 1$

c. $f(x) = \cot x$

$x = \pi n, n \in \mathbb{Z}$

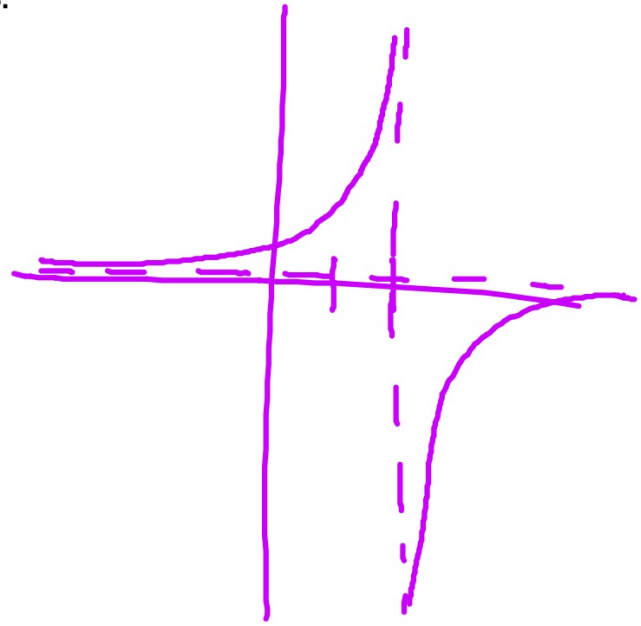
Determine whether f(x) approaches infinity or negative infinity from the right and from the left of the vertical asymptotes.

③ $g(x) = \dfrac{-1}{x-2}$

VA $x = 2$

$\lim_{x \to 2^-} g(x) \quad \infty$

$\lim_{x \to 2^+} g(x) \quad -\infty$



$\underline{\tan x}$

$x = \frac{\pi}{2} + \pi n, n \in Z$

$\underline{\cot x}$

$x = \pi n, n \in Z$

$\underline{\sec x}$

$x = \frac{\pi}{2} + \pi n, n \in Z$

$\underline{\csc x}$

$x = \pi n, n \in Z$

VA for trig functions

Find all vertical asymptotes.

Ex 4

$$h(t) = \frac{t^2 - 2t}{t^4 - 16}$$

$$= \frac{t(t-2)}{(t^2+4)(t^2-4)}$$

$$= \frac{t(t-2)}{(t^2+4)(t-2)(t+2)}$$

VA: $t = -2$

Ex 5

$$h(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2}$$

$$h(x) = \frac{(x+2)(x-2)}{x^2(x+2) + 1(x+2)}$$

$$= \frac{(x+2)(x-2)}{(x^2+1)(x+2)}$$

$$h(x) = \frac{x-2}{x^2+1}$$

NO VA