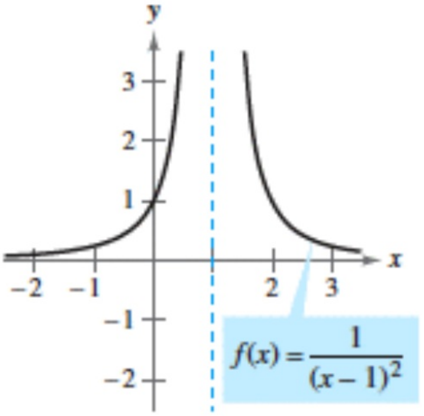
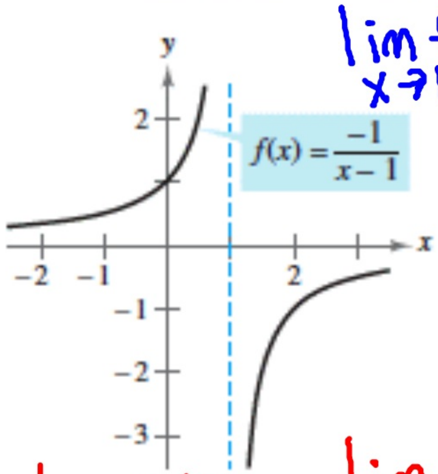
1.5 Infinite Limits

- Determine infinite limits from the left and from the right.
- Find and sketch the vertical asymptotes of the graph of a function.



$\lim_{x \to 1} f(x)$

∞



$\lim_{x \to 1} f(x)$

dne

$\lim_{x \to 1^-} f(x)$ $\lim_{x \to 1^+} f(x)$

$\infty \neq -\infty$

DEFINITION OF VERTICAL ASYMPTOTE

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f.

Determine all vertical asymptotes of the graph of each function.

a. $f(x) = \dfrac{1}{2(x+1)}$ **b.** $f(x) = \dfrac{x^2+1}{x^2-1}$ **c.** $f(x) = \cot x$

VA: $X = -1$

VA: $x = \pm 1$

$X = 0, \pi, 2\pi \ldots$

$X = \pi n, \; n \in \mathbb{Z}$

Find all vertical asymptotes.

Ex 1 $h(t) = \frac{t^2 - 2t}{t^4 - 16} = \frac{t\cancel{(t-2)}}{(t^2+4)(t+2)\cancel{(t-2)}}$

$t = -2$

Ex 2 $h(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2}$

$x^2(x+2)+1(x+2)$

$(x^2+1)(x+2)$

$= \frac{\cancel{(x+2)}(x-2)}{(x^2+1)\cancel{(x+2)}}$

No VA

THEOREM 1.15 PROPERTIES OF INFINITE LIMITS

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x\to c} f(x) = \infty \qquad \text{and} \qquad \lim_{x\to c} g(x) = L.$$

1. Sum or difference: $\lim_{x\to c} [f(x) \pm g(x)] = \infty$
2. Product: $\lim_{x\to c} [f(x)g(x)] = \infty, \quad L > 0$

   $\lim_{x\to c} [f(x)g(x)] = -\infty, \quad L < 0$
3. Quotient: $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$

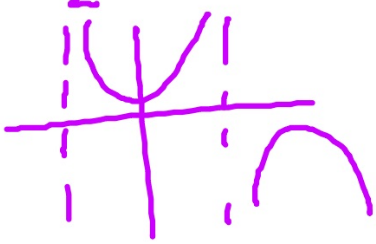
Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.

Ex 3

$$\lim_{x\to(\pi/2)^+} \frac{-2}{\cos x}$$

$$\lim_{x\to\frac{\pi}{2}^+} -2\sec x$$

$$-2\lim_{x\to\frac{\pi}{2}^+} \sec x = -2(-\infty)$$
$$= \infty$$



Ex 4

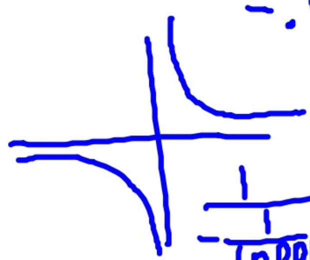
$$\lim_{x\to0^-}\left(x^2 - \frac{1}{x}\right)$$

$$\lim_{x\to0^-} x^2 - \lim_{x\to0^-} \frac{1}{x}$$

$$0 - (-\infty)$$

$$\infty$$

$$\frac{1}{-.0001}$$



$$\frac{1}{-\frac{1}{10000}}$$

$$-10000$$

p. 88

13 - 27 odd

37 - 49 odd