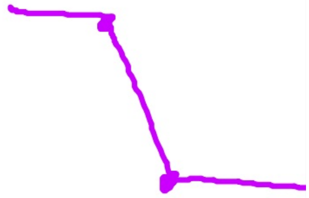
1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

$$f(x) = \begin{cases} 2 & x \le -1 \\ ax+b & -1 < x < 3 \\ -2 & x \ge 3 \end{cases}$$



$$\lim_{x \to -1^-} 2 = \lim_{x \to -1^+} (ax+b) = f(-1)$$

$$2 = -a+b$$

$$\lim_{x \to 3^-} (ax+b) = \lim_{x \to 3^+} -2 = f(3)$$

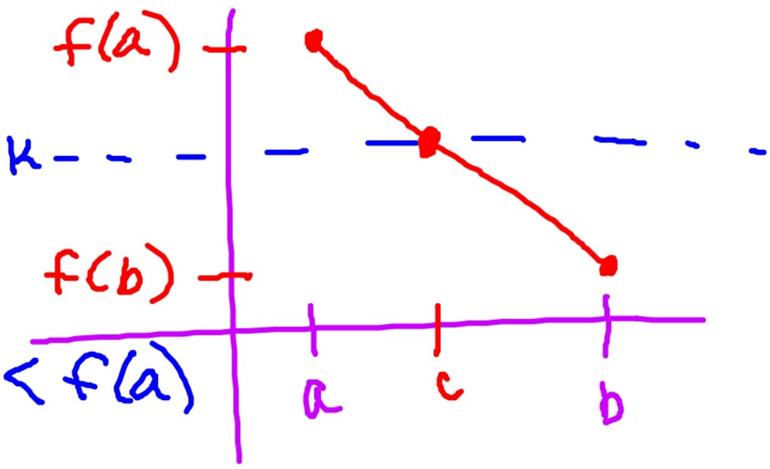
$$3a+b = -2$$

THEOREM 1.13 INTERMEDIATE VALUE THEOREM

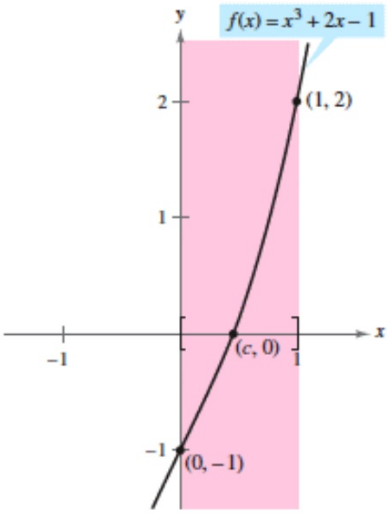
If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$

IVT



$f(b) < k < f(a)$

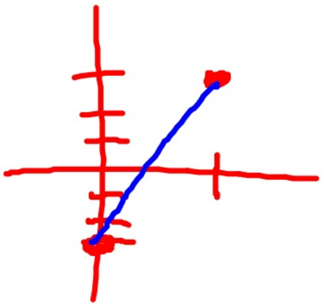

$f(x) = x^3 + 2x - 1$
y
x
2
1
−1
−1
1
(1, 2)
(c, 0)
(0, −1)

Use IVT to prove that there will be a value c on the closed interval such that f(c) = 0

$$f(x) = x^3 + 5x - 3 \qquad [\overset{a}{0}, \overset{b}{1}]$$

$f(0) = -3$

$f(1) = 3$



Since f(x) is continuous on [0, 1] and $f(0) < 0 < f(1)$, by IVT there exists at least one value c such that $f(c) = 0$

Prove that there is a value c guaranteed by IVT. Then, find c.

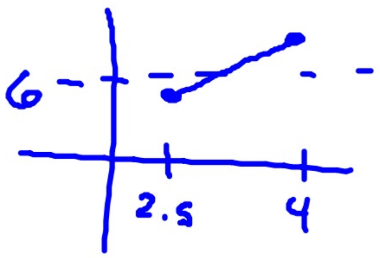
$$f(x) = \frac{x^2 + x}{x - 1}, \quad \left[\frac{5}{2}, 4\right], \quad f(c) = 6$$

$f(x)$ cont. $\left[\frac{5}{2}, 4\right]$

Since f(x) is continuous on [2.5, 4] and f(2.5) < 6 < f(4), by IVT there exists at least one value c such that f(c) = 6

$f(2.5) = 5.8\overline{3}$

$f(4) = \frac{20}{3} = 6\frac{2}{3}$



$$\frac{6}{1} = \frac{c^2 + c}{c - 1} \qquad c = \cancel{2}, 3$$

$$6c - 6 = c^2 + c$$

$$0 = c^2 - 5c + 6$$

$$0 = (c-2)(c-3)$$

One sided limits

types of discontinuities (removable/nonremovable)

Find a and b so that f(x) is continuous
(use the definition of continuity; use limits)

IVT

p.80 – 81
83, 84
91, 92

graphs

- trig functions with asymptotes
- $y = \frac{|x|}{x}$ or a translation

63. $f(1) = 3$

Find a so that $\lim_{x\to 1^-}(ax - 4) = 3$

$$a(1) - 4 = 3$$

$$a = 7.$$

64. $f(1) = 3$

Find a so that $\lim_{x\to 1^+}(ax + 5) = 3$

$$a(1) + 5 = 3$$

$$a = -2.$$

65. $f(2) = 8$

Find a so that $\lim_{x\to 2^+} ax^2 = 8 \Rightarrow a = \frac{8}{2^2} = 2.$

66. $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \frac{4\sin x}{x} = 4$

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (a - 2x) = a$

Let $a = 4$.

67. Find a and b such that $\lim_{x \to -1^+} (ax + b) = -a + b = 2$ and $\lim_{x \to 3^-} (ax + b) = 3a + b = -2$.

$$\begin{array}{rl} a - b &= -2 \\ \underline{(+)3a + b} &\underline{= -2} \\ 4a &= -4 \\ a &= -1 \\ b &= 2 + (-1) = 1 \end{array}$$

$$f(x) = \begin{cases} 2, & x \le -1 \\ -x + 1, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$$