1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

Ex 1 **What value of a will make f(x) continuous?** — def. continuity

$$f(x) = \begin{cases} 3x^3, & x \le 1 \\ ax + 5, & x > 1 \end{cases}$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$\lim_{x \to 1^-} 3x^3 = \lim_{x \to 1^+} (ax+5)$$

$$3 = a + 5$$

$$-2 = a$$

Ex 2

$$g(x)=\begin{cases} x^2-c & x<5 \\ 4x+2c & x\geq 5 \end{cases}$$

$$\lim_{x\to 5^-} g(x) = \lim_{x\to 5^+} g(x) = g(5)$$

$$\lim_{x\to 5^-}(x^2-c) = \lim_{x\to 5^+}(4x+2c) = g(5)$$

$$25-c = 20+2c$$

$$5/3 = c$$

3] $h(x) = \begin{cases} 5xa+2 & x \neq -3 \\ -11 & , x=-3 \end{cases}$ ← $(-3,-11)$

$\lim_{x \to -3} h(x) = h(-3)$

$\lim_{x \to -3} (5xa+2) = -11$

$-15a+2 = -11$

$-15a = -13$

$a = \frac{13}{15}$

Ex 3

$$f(x) = \begin{cases} Ax - B\,, & \text{if } x \le -1 \\ 2x^2 + 3Ax + B\,, & \text{if } -1 < x \le 1 \\ 4\,, & \text{if } x > 1 \end{cases}$$

$x = -1$ $\qquad$ $x = 1$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$

$\lim_{x \to -1^-} (Ax - B) = \lim_{x \to -1^+} (2x^2 + 3Ax + B)$

$-A - B = 2 - 3A + B$

$2A - 2B = 2$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\lim_{x \to 1^-} (2x^2 + 3Ax + B) = \lim_{x \to 1^+} 4$

$2 + 3A + B = 4$

$3A + B = 2$

$$\begin{array}{r} A - B = 1 \\ +\ 3A + B = 2 \\ \hline 4A = 3 \end{array}$$

$A = \frac{3}{4}$ $\qquad$ $B = -\frac{1}{4}$

p. 80

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