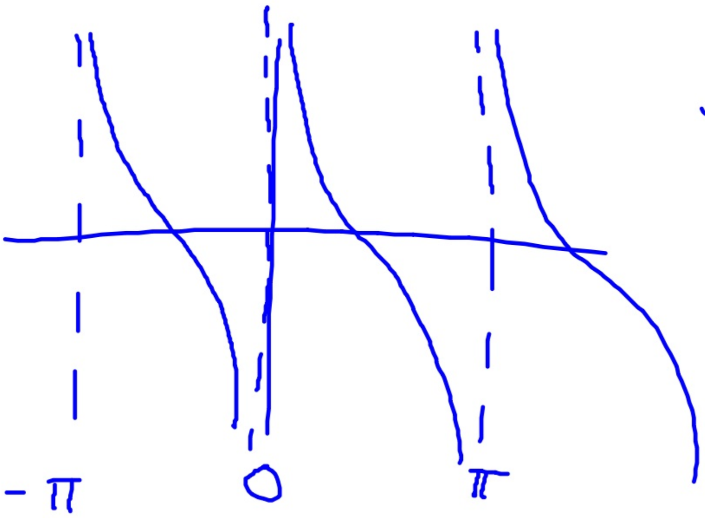
21.) $\lim_{x \to \pi} \cot x$ dne



$\lim_{x \to \pi^-} \cot x \quad -\infty$

$\lim_{x \to \pi^+} \cot x \quad +\infty$

23.) $\lim_{x \to 4^-} (5[x] - 7)$

$\lim_{x \to 4^-} 5[x] - \lim_{x \to 4^-} 7$

$5[3.9] - 7$

$5(3) - 7$

8

1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

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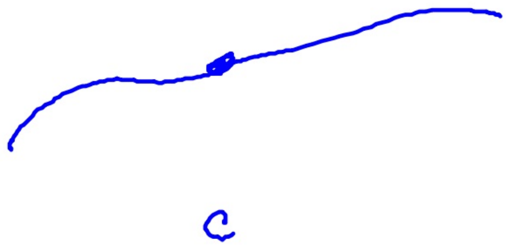
DEFINITION OF CONTINUITY

Continuity at a Point: A function f is **continuous at c** if the following three conditions are met.

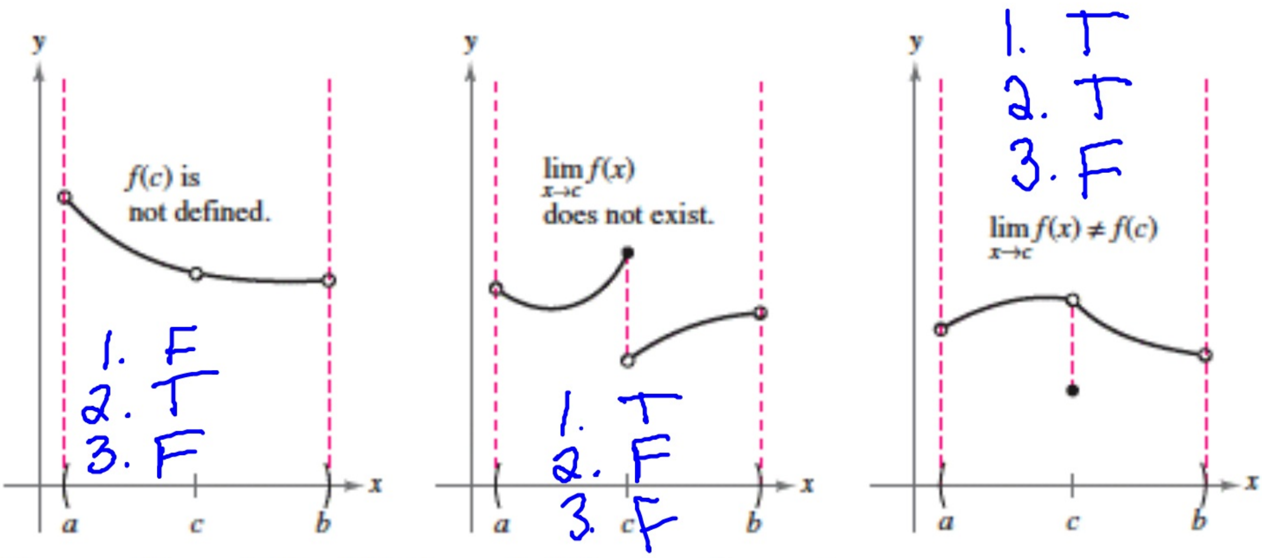
1. $f(c)$ is defined.

2. $\lim\limits_{x \to c} f(x)$ exists.

* **3.** $\lim\limits_{x \to c} f(x) = f(c)$



Continuity on an Open Interval: A function is **continuous on an open interval (a, b)** if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is **everywhere continuous.**



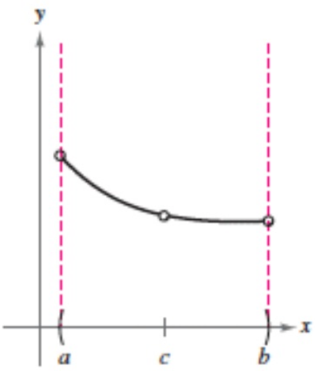
Three conditions exist for which the graph of f is not continuous at $x = c$.

In Figure 1.25, it appears that continuity at $x = c$ can be destroyed by any one of the following conditions.

1. The function is not defined at $x = c$.
2. The limit of $f(x)$ does not exist at $x = c$.
3. The limit of $f(x)$ exists at $x = c$, but it is not equal to $f(c)$.

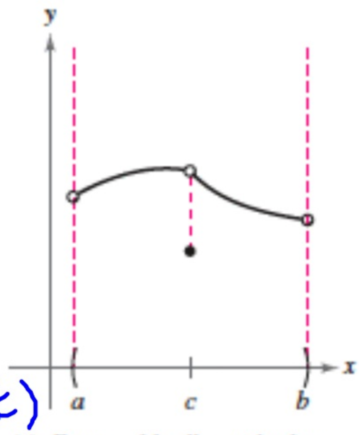
Removable Discontinuity:

If moving or adding one point can 'fix' the discontinuity, this is called removable.



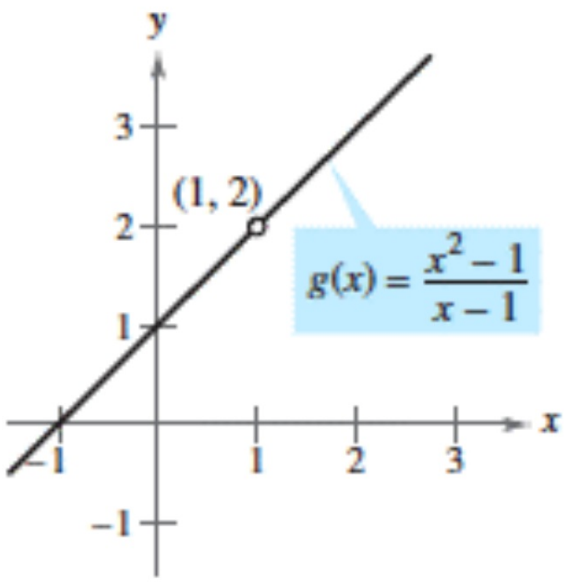
(a) Removable discontinuity

$\lim_{x \to c} f(x)$ exists but $\lim_{x \to c} f(x) \neq f(c)$



(c) Removable discontinuity

For removable, the general limit will exist but f(c) will not equal that limit.



(b) Removable discontinuity at $x = 1$

$$g(x) = \frac{x^2 - 1}{x - 1}$$

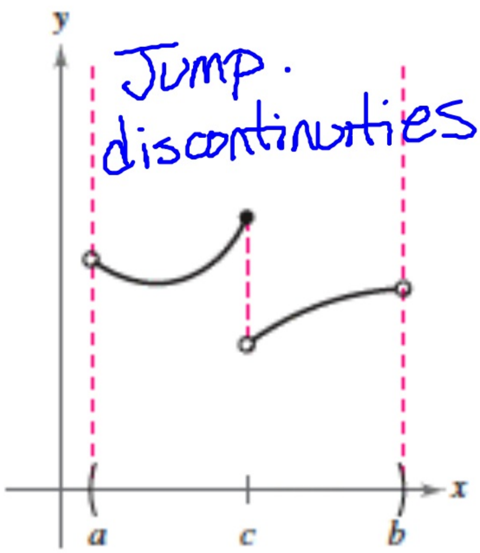
$$= \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

$$g(x) = x + 1$$

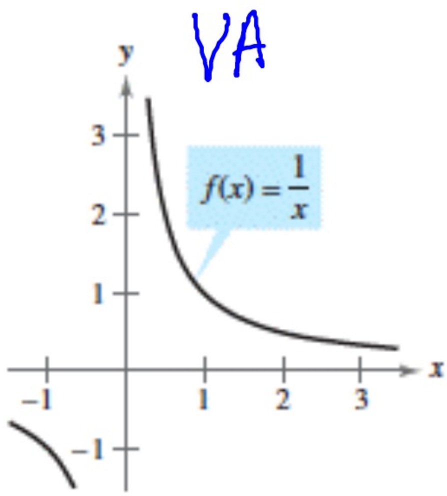
hole@ (1, 2)

Nonremovable discontinuity:

If 'moving' more than one point is necessary to fix the discontinuity



(b) Nonremovable discontinuity



(a) Nonremovable discontinuity at $x = 0$

$\lim_{x \to c} f(x)$ dne

THEOREM 1.11 PROPERTIES OF CONTINUITY

If b is a real number and f and g are continuous at $x = c$, then the following functions are also continuous at c.

1. Scalar multiple: bf
2. Sum or difference: $f \pm g$
3. Product: fg
4. Quotient: $\dfrac{f}{g}$, if $g(c) \neq 0$

The following types of functions are continuous at every point in their domains.

1. Polynomial: $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1 x + a_0$
2. Rational: $r(x) = \dfrac{p(x)}{q(x)}$, $q(x) \neq 0$
3. Radical: $f(x) = \sqrt[n]{x}$
4. Trigonometric: $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\csc x$

Discuss the continuity.

Ex 1 $f(x) = \dfrac{x-6}{x^2-36}$

$$f(x) = \frac{\cancel{x-6}}{(x+6)\cancel{(x-6)}}$$

$$f(x) = \frac{1}{x+6}$$

Removable discontinuity
at x = 6

Nonremovable discontinuity
at x = -6

2

$$f(x) = \frac{x-1}{x^2+x-2}$$

$$\frac{\cancel{x-1}}{(x+2)\cancel{(x-1)}}$$

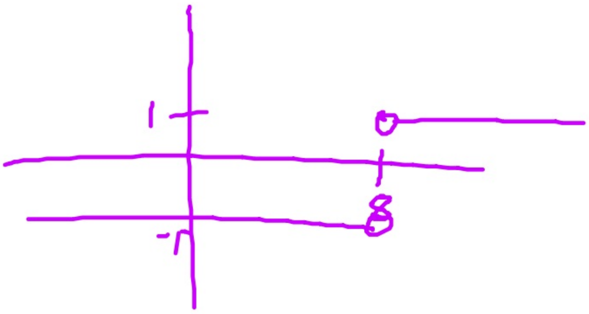
$$\frac{1}{x+2}$$

Removable discontinuity at x = 1

Nonremovable discontinuity at x = -2

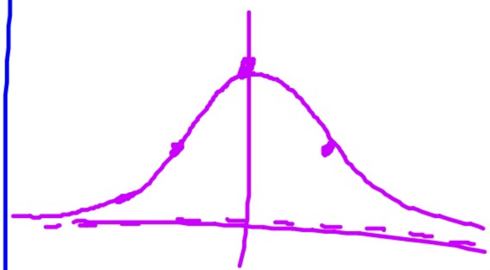
③ $f(x) = \frac{|x-8|}{x-8}$

Nonremovable discontinuity at x = 8



④ $f(x) = \frac{5}{x^2+1}$

f(x) is continuous everywhere



x	y
0	5
1	2.5
-1	2.5

5 $g(x) = \begin{cases} -x^2+4x+1, & x \le 2 \\ 3x+7, & x > 2 \end{cases}$

$$\lim_{x \to 2^-} (-(x)^2+4x+1) \neq \lim_{x \to 2^+} (3x+7)$$

5 13

Nonremovable discontinuity at x = 2