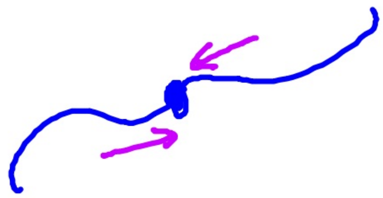
1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

Ex 1 What value of a will make f(x) continuous?

$$f(x) = \begin{cases} 3x^3, & x \le 1 \\ ax + 5, & x > 1 \end{cases}$$



1) Start by writing the definition of continuity.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

2) Evaluate the limits and find the missing variable(s)

$$\lim_{x \to 1^-} 3x^3 = \lim_{x \to 1^+} (ax+5)$$

$$3 = a + 5$$

$$-2 = a$$

Ex 2

$$g(x) = \begin{cases} x^2 - c & x < 5 \\ 4x + 2c & x \geq 5 \end{cases}$$

$$\lim_{x \to 5^-} g(x) = \lim_{x \to 5^+} g(x) = g(5)$$

$$\lim_{x \to 5^-} (x^2 - c) = \lim_{x \to 5^+} (4x + 2c)$$

$$25 - c = 20 + 2c$$

$$5/3 = c$$

Ex 3

$$f(x) = \begin{cases} Ax - B\ , & \text{if } x \le -1 \\ 2x^2 + 3Ax + B\ , & \text{if } -1 < x \le 1 \\ 4\quad , & \text{if } x > 1 \end{cases}$$

$$\lim_{x \to -1^-} Ax - B = \lim_{x \to -1^+} 2x^2 + 3Ax + B = f(-1)$$

$$-A - B = 2 - 3A + B\ ;\ 2A - 2B = 2$$

$$A - B = 1$$

$$\lim_{x \to 1^-} 2x^2 + 3AX + B = \lim_{x \to 1^+} 4 = f(1)$$

$$2 + 3A + B = 4\ ;\ 3A + B = 2$$

$$A - B = 1$$
$$3A + B = 2$$
$$4A = 3$$
$$A = 3/4$$
$$B = -1/4$$

$$f(x)=\begin{cases}2x+a, & x\le -1\\ x^2+1, & -1<x\le 2\\ bx-1, & x>2\end{cases}$$

$$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^+} f(x)=f(-1)$$

$$-2+a=2 \quad ; a=4$$

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=f(2)$$

$$5=2b-1$$

$$3=b$$

$$g(x)=\begin{cases}\dfrac{x^2-a^2}{x-a}, & x\neq a\\[6pt] 8 & x=a\end{cases}$$

$(a,8)$

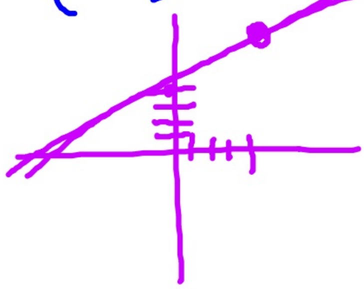
$$\lim_{x\to a} g(x) = g(a)$$

$$\lim_{x\to a}\frac{x^2-a^2}{x-a}$$

$$\lim_{x\to a} \cancel{x}_{a}+a = 8$$

$$2a=8;\ a=4$$

$$g(x)=\begin{cases}x+4, & x\neq 4\\ 8, & x=4\end{cases}$$

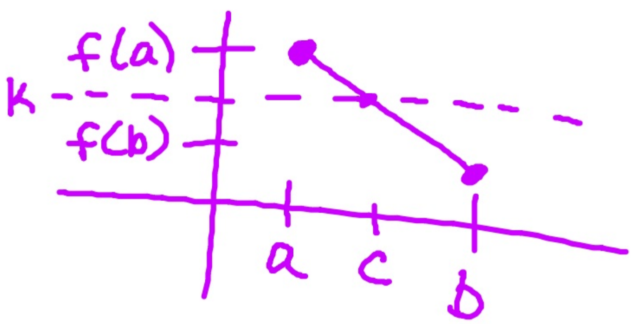


THEOREM 1.13 INTERMEDIATE VALUE THEOREM

If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$

IVT



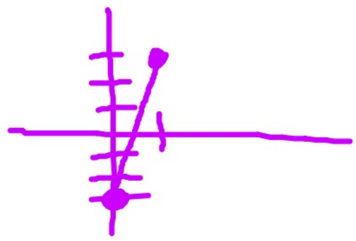
dvh theorem

Use IVT to prove that there will be a value c on the closed interval such that f(c) = 0

$$f(x) = x^3 + 5x - 3 \qquad [0, 1]$$

$f(0) = -3$

$f(1) = 3$



Since f(x) is continuous on [0,1], and f(0) < 0 < f(1), by IVT there must exist a value c such that f(c) = 0.

Prove that there is a value c guaranteed by IVT. Then, find c.

$$f(x) = \frac{x^2 + x}{x - 1}, \quad \left[\frac{5}{2}, 4\right], \quad f(c) = 6$$

$f\left(\frac{5}{2}\right) = \frac{35}{6} = 5\frac{5}{6}$

$f(4) = 6\frac{2}{3}$

Since f(x) is continuous on [2.5, 4] and f(2.5) < 6 < f(4), by IVT there exists a value c such that f(c) = 6.

$$\frac{6}{1} = \frac{c^2 + c}{c - 1}$$

$6c - 6 = c^2 + c$

$0 = c^2 - 5c + 6$

$(c-2)(c-3)$

$c = \cancel{2}, 3$

115.) $\lim_{x \to c^-} (1-x^2) = \lim_{x \to c^+} (x) = f(c)$

$a=1$
$b=1$
$c=-1$

$1-c^2 = c$

$c^2+c-1=0$

$$\frac{-1 \pm \sqrt{1-4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$