1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the Squeeze Theorem.

45.) $$\lim_{x \to -1} \frac{x^2 - 1}{x + 1} = \lim_{x \to -1} \frac{(x+1)(x-1)}{x+1}$$

$$= \lim_{x \to -1} \frac{x-1}{1} = -2$$

61.) $\lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 2x}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\cancel{2x} + 2\Delta x \cancel{- 2x}}{\Delta x}$

let $\Delta x = h$

$\lim_{\Delta x \to 0} 2$

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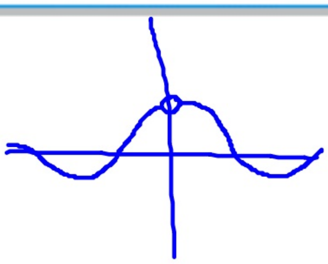
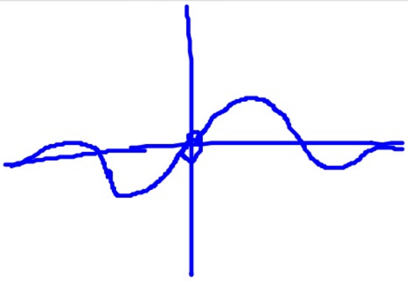
Strategy for finding limits

1. Direct Substitution
2. Algebraic techniques
(factoring or rationalizing or simplifying)
3. Special Cases

THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$ **2.** $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$





Ex 1

$$\lim_{x\to 0} \frac{3(1-\cos x)}{x}$$

$$3\left[\lim_{x\to 0} \frac{1-\cos x}{x}\right]$$

$$3\cdot 0$$

$$0$$

Ex 2

$$\lim_{t\to 0} \frac{\sin 3t}{2t}$$

$$\frac{1}{2}\lim_{t\to 0} \frac{\sin 3t}{t}\cdot\frac{3}{3}$$

$$\frac{3}{2}\left[\lim_{t\to 0} \frac{\sin 3t}{3t}\right]$$

$$\frac{3}{2}\cdot 1 = \frac{3}{2}$$

$$\lim_{x\to 0} \frac{\sin 4x}{11x} = \frac{1}{11}\lim_{x\to 0} \frac{\sin 4x}{x} \cdot \frac{4}{4}$$

$$\frac{4}{11}\left[\lim_{x\to 0} \frac{\sin 4x}{4x}\right]$$

$$\frac{4}{11} \cdot 1$$

$$\frac{4}{11}$$

Ex 4: Given $f(x) = 5x - 2$, $f(x+h) = 5(x+h) - 2$

find $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$. let $\Delta x = h$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5(x+h) - 2 - (5x-2)}{h}$$

$$= \lim_{h \to 0} \frac{5x + 5h - 2 - 5x + 2}{h} = \lim_{h \to 0} \frac{5h}{h} = \lim_{h \to 0} 5$$

$$= 5$$

$$(x+h)^2 = x^2 + 2xh + h^2$$

$$3(x+h)^2 = 3(x^2 + 2xh + h^2)$$

59.) $\lim_{x \to 0} \dfrac{\frac{1 \cdot 3(3+x)}{3+x} - \frac{1 \cdot 3(3+x)}{3}}{x \cdot 3(3+x)}$ LCD: $3(3+x)$

$\lim_{x \to 0} \dfrac{3 - (3+x)}{3x(3+x)}$

$\lim_{x \to 0} \dfrac{3 - 3 - x}{3x(3+x)}$

$\lim_{x \to 0} \dfrac{-x}{3x(3+x)}$

$\lim_{x \to 0} \dfrac{-1}{3(3+x)}$

$-\dfrac{1}{9}$

$f(x) = 2x^2 + 3x - 1 \quad f(x+h) = 2(x+h)^2 + 3(x+h) - 1$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 + 3(x+h) - 1 - (2x^2 + 3x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\overset{2x^2 + 4xh + 2h^2}{2(x^2 + 2xh + h^2)} + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}$$

$$\lim_{h \to 0} \frac{4xh + 2h^2 + 3h}{h} = \lim_{h \to 0} \frac{h(4x + 2h + 3)}{h}$$

$$= 4x + 3$$