1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the Squeeze Theorem.

THEOREM 1.1 SOME BASIC LIMITS

Let b and c be real numbers and let n be a positive integer.

1. $\lim_{x \to c} b = b$
2. $\lim_{x \to c} x = c$
3. $\lim_{x \to c} x^n = c^n$

THEOREM 1.2 PROPERTIES OF LIMITS

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

Strategy for finding limits

1. Direct Substitution
2. Algebraic techniques
(factoring or rationalizing or simplifying)
3. Special Cases

THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS

1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$ **2.** $\lim_{x\to 0} \frac{1 - \cos x}{x} = 0$

Ex 1

$$\lim_{x\to 2}\frac{\sqrt{x+2}}{x-4}$$

$$\frac{\sqrt{2+2}}{2-4}$$

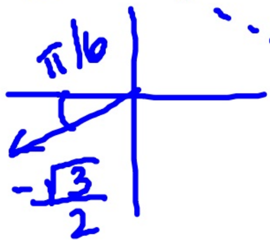
$$\frac{2}{-2}$$

$$-1$$

Ex 2

$$\lim_{x\to 7}\sec\left(\frac{\pi x}{6}\right)$$

$$\cos\frac{7\pi}{6}$$



$$\sec\frac{7\pi}{6}$$

$$-\frac{2}{\sqrt{3}}$$

or

$$-\frac{2\sqrt{3}}{3}$$

Ex 3

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(2x-3)}{\cancel{(x+1)}}$$

$$\lim_{x \to -1} 2x - 3$$

$$-5$$

Ex 4

$$\lim_{x \to -1} \frac{(x^3 + 1)}{x + 1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(x^2 - x + 1)}{\cancel{x+1}}$$

$$(-1)^2 - (-1) + 1$$

$$1 + 1 + 1$$

$$3$$

Ex 5

$$\lim_{x\to 3} \frac{(\sqrt{x+1}-2)}{x-3} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{\cancel{x+1-4}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{1}{\sqrt{x+1}+2}$$

$$\frac{1}{4}$$

Ex 6

$$\lim_{x\to 0}\frac{[1/(x+4)]-(1/4)}{x} = \lim_{x\to 0}\frac{\left(\frac{1}{x+4}-\frac{1}{4}\right)\cdot 4(x+4)}{\frac{x}{1}\cdot 4(x+4)}$$

$$\lim_{x\to 0}\frac{4-(x+4)}{4x(x+4)}$$

$$\lim_{x\to 0}\frac{-\cancel{x}}{4\cancel{x}(x+4)}$$

$$\lim_{x\to 0}\frac{-1}{4(x+4)} = \frac{-1}{16}$$

⑦ $\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - \cancel{x^2}}{\Delta x}$

$(x+\Delta x)(x+\Delta x)$

$x^2 + \underline{x\Delta x} + \underline{x\Delta x} + \Delta x^2$

$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}}$

$\lim_{\Delta x \to 0} (2x+\Delta x)$

(Δx)

$2x$

Ex 9

$$\lim_{x\to 0} \frac{3(1-\cos x)}{x}$$

$$3\left[\lim_{x\to 0} \frac{1-\cos x}{x}\right]$$

$$3 \cdot 0$$

$$0$$

Ex 10

$$\lim_{t\to 0} \frac{\sin(3t)}{2t}$$

$$\frac{1}{2} \lim_{t\to 0} \frac{\sin 3t}{t} \cdot \frac{3}{3}$$

$$\frac{3}{2}\left[\lim_{t\to 0} \frac{\sin 3t}{3t}\right]$$

$$\frac{3}{2} \cdot 1 = \frac{3}{2}$$

Ex 11: Given f(x) = 5(x) - 2,

find $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

$f(x+h) = 5(x+h) - 2$

$$\lim_{h \to 0} \frac{5(x+h) - 2 - (5x - 2)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{5x} + 5h \cancel{-2} \cancel{-5x} \cancel{+2}}{h}$$

$$\lim_{h \to 0} \frac{5\cancel{h}}{\cancel{h}} = 5$$

12 $f(x) = 3x^2 + 4x - 1$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 6x + 4$$

23c.) $\lim_{x \to 1} g(f(x)) = \lim_{x \to 1} (5-x)^3$

$f(x) = 5 - x$

$g(x) = x^3$

$(5-1)^3$

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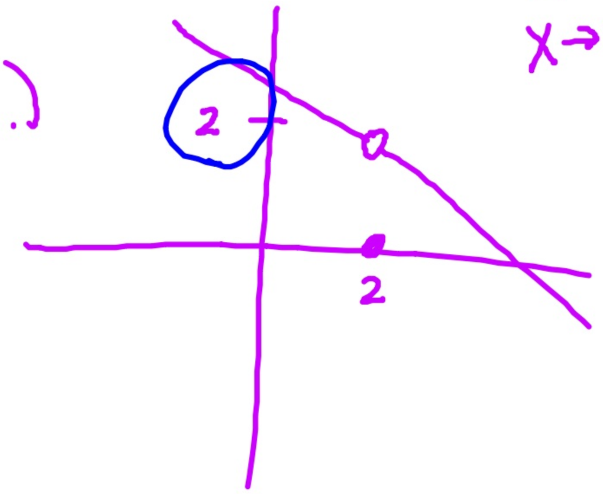
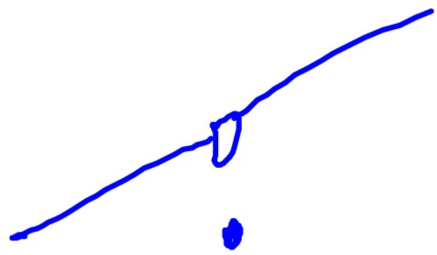
17.)



$$\lim_{x \to 2} f(x) = 2$$



47.) $\lim_{x \to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}}$

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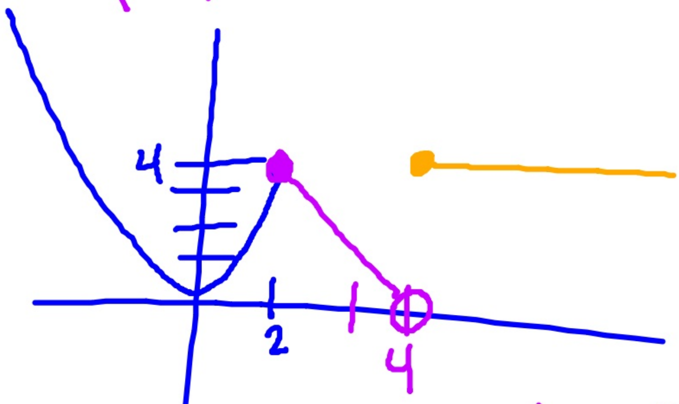
$$A^3 - B^3 = (A-B)(A^2+AB+B^2)$$

29.) $f(x) = \begin{cases} x^2 & x \le 2 \\ 8-2x & 2 < x \ 4 \\ 4 & x \ge 4 \end{cases}$

$\lim_{x \to c} f(x)$ exists?



Limit exists everywhere except $x=4$