1.2 Finding Limits Graphically and Numerically

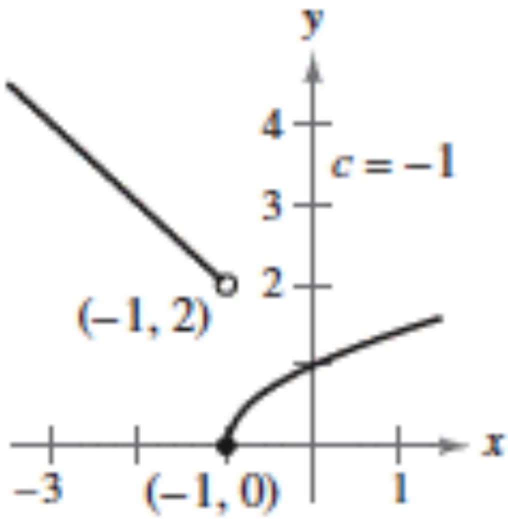
- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.

1. Numerical approach — Construct a table of values.
2. Graphical approach — Draw a graph by hand or using technology.
3. Analytic approach — Use algebra or calculus.

COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

DNE
Type #1



$$\lim_{x \to -1^-} f(x) = 2$$

$$\lim_{x \to -1^+} f(x) = 0$$

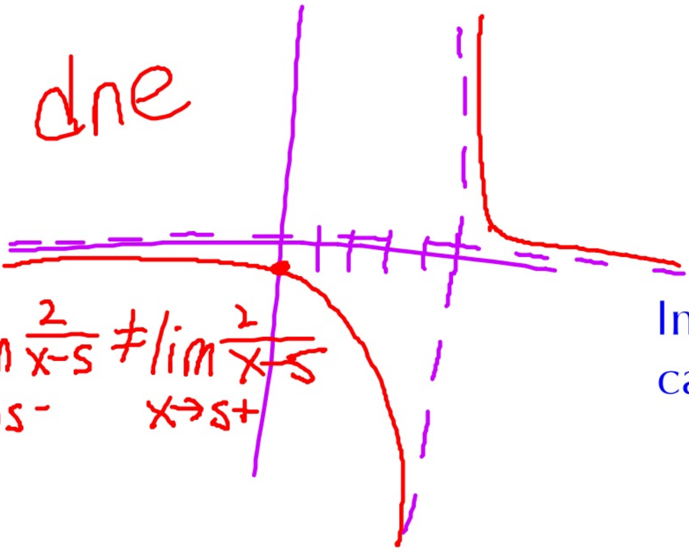
DNE

$$\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$$

DNE Type #2

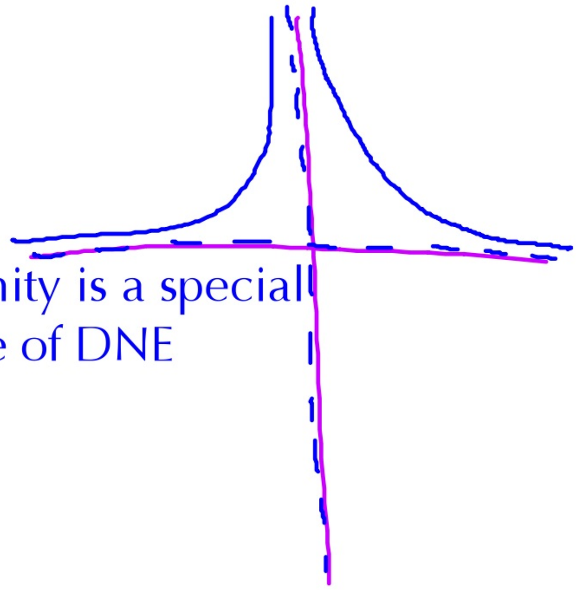
a.) $\lim_{x\to5}\frac{2}{x-5}$

dne



$\lim_{x\to5^-}\frac{2}{x-5} \neq \lim_{x\to5^+}\frac{2}{x-5}$

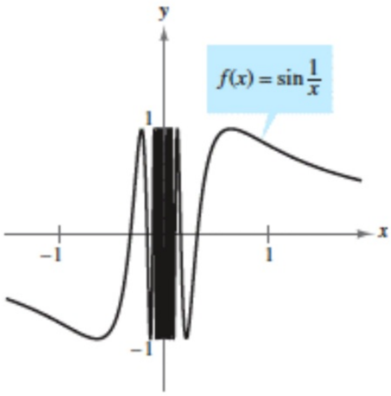
b.) $\lim_{x\to0}\frac{1}{x^2}$ ∞



Infinity is a special case of DNE

DNE Type #3

3.



$\lim_{x \to 0} f(x)$ does not exist.

$f(x) = \sin \frac{1}{x}$

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	.544	.506	-.827	.827	-.506	-.544

(a) $f(-2)$ undefined

(b) $\lim_{x \to -2} f(x)$ dne

(c) $f(0) = 4$

(d) $\lim_{x \to 0} f(x)$ dne

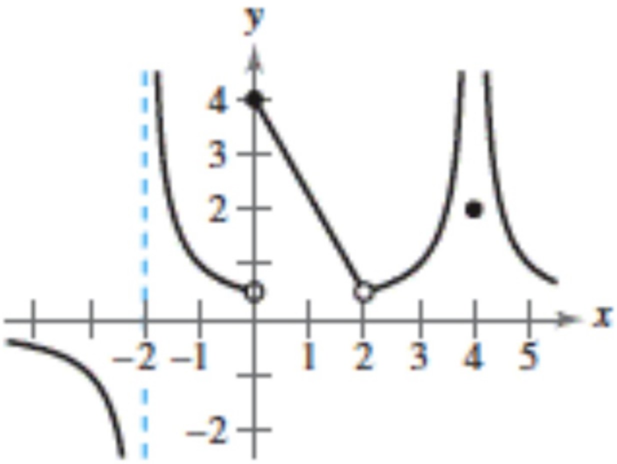
(e) $f(2)$ undefined

(f) $\lim_{x \to 2} f(x) = 1/2$

(g) $f(4) = 2$

(h) $\lim_{x \to 4} f(x)$ ∞ or dne

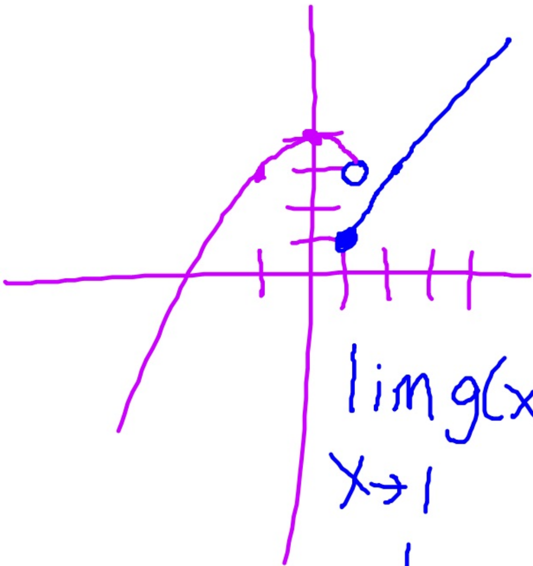
more descriptive



Sketching Piecewise functions

Sketch the graph of the following piecewise function.

$$g(x) = \begin{cases} -x^2+4 & \text{if } x < 1 \\ 2x-1 & \text{if } x \geq 1 \end{cases}$$



$\lim_{x \to 1} g(x)$

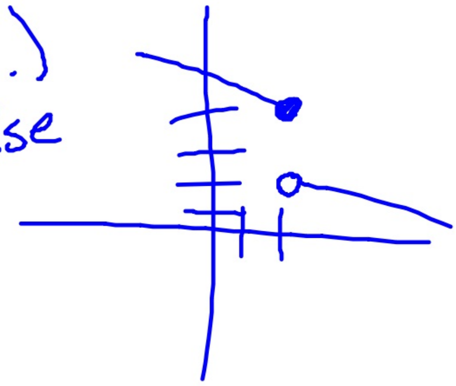
dne

$-x^2+4$	$2x-1$
$(1,3)$ ∘	$(1,1)$ •
$(0,4)$ •	
$(-1,3)$ •	

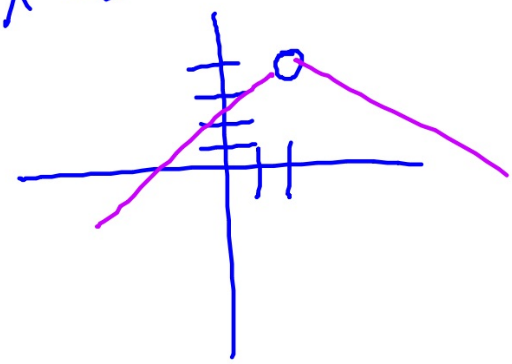
(a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

(b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

a.) False

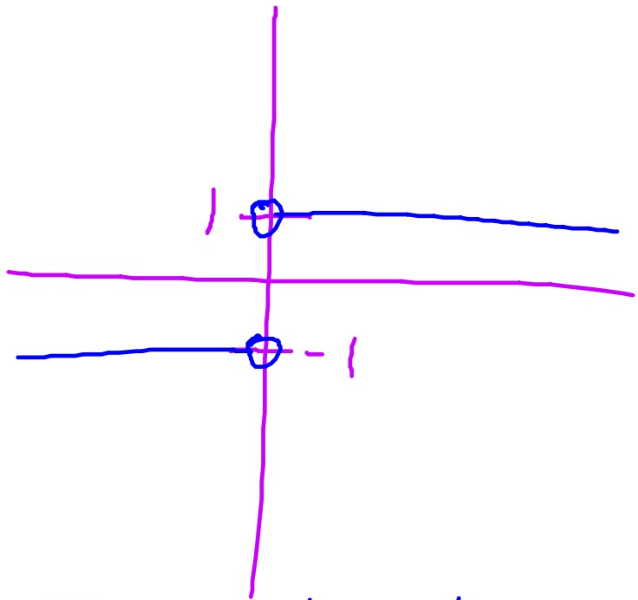


b.) $\lim_{x \to 2} f(x) = 4$ False



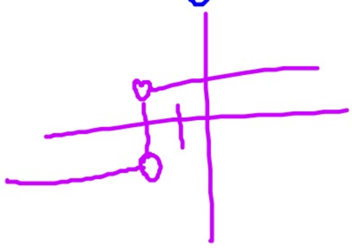
$$f(x) = \frac{|x|}{x}$$

x	$f(x)$
-2	-1
-1	-1
0	und.
1	1
2	1
3	1



$$* \; g(x) = \frac{|x+2|}{x+2}$$

left 2



$$y = a|x-h| + K$$

a : vertical stretch/shrink
faces up or down

h : horizontal shift

K : vertical shift

p.55: 15-31odd

$$y = -2|x+3| - 5$$

h : left 3

K : down 5

a : vert. stretch
faces down