1.2 Finding Limits Graphically and Numerically

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.

1. Numerical approach	Construct a table of values.
2. Graphical approach	Draw a graph by hand or using technology.
3. Analytic approach	Use algebra or calculus.

Ex 1

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = .25$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	.256	.2506	.25006	.24994	.2494	.2439

Ex 2

$$\lim_{x \to 4} \frac{[x/(x+1)] - (4/5)}{x - 4} = .04$$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	.0408	.0401	.04001	.03990	.03992	.0392

$$\lim_{x\to 4}\left(\frac{\frac{x}{x+1}-\frac{4}{5}}{\frac{x-4}{1}}\right)\cdot\frac{5(x+1)}{5(x+1)}$$

$$\lim_{x\to 4}\frac{5x-4(x+1)}{(x-4)\cdot 5(x+1)}$$

$$\lim_{x\to 4}\frac{\cancel{x-4}}{\cancel{(x-4)}\cdot 5(x+1)}$$

$$\lim_{x\to 4}\frac{1}{5(x+1)}=\frac{1}{25}$$

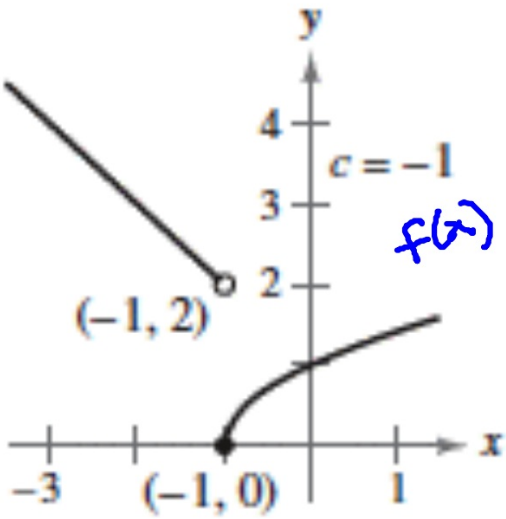
COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

DNE : does not exist

∞ or $-\infty$ are special cases of DNE

DNE
Type #1



$\lim_{x \to -1} f(x)$ does not exist

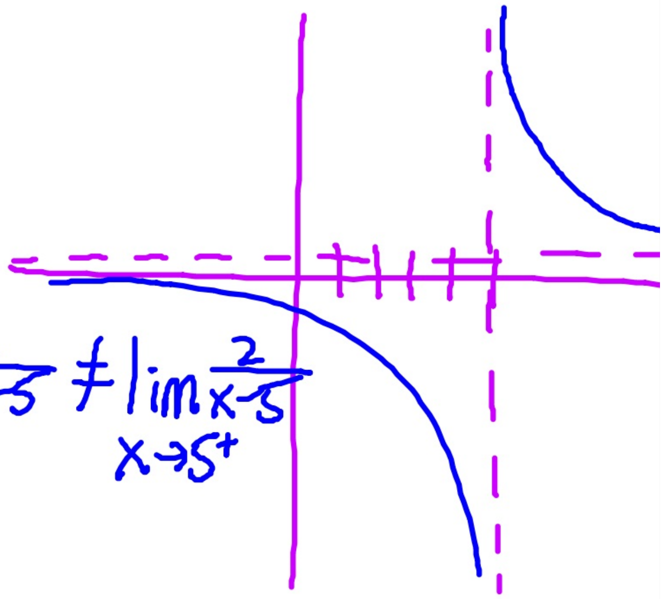
Justify

$\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$

$2 \neq 0$

DNE Type #2

2. $$\lim_{x\to 5} \frac{2}{x-5}$$



$$\lim_{x\to 5^-} \frac{2}{x-5} \neq \lim_{x\to 5^+} \frac{2}{x-5}$$

VA: $x-5=0$

$x=5$

HA: $y=0$ "Bobo"

Bobo Botn Eats d.c.

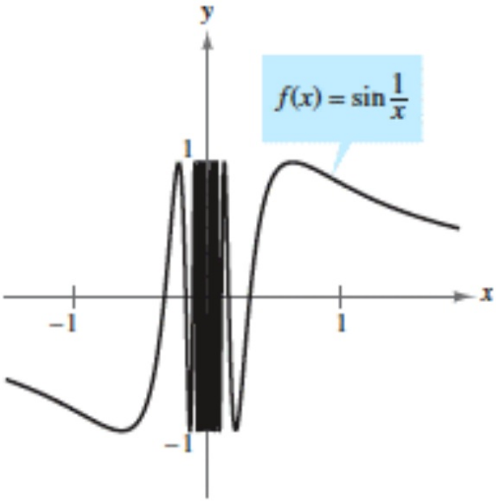
DNE Type #3

$f(x) = \sin\frac{1}{x}$

does not exist

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

3.



$\lim_{x \to 0} f(x)$ does not exist.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	$-.54$	$.506$	$-.83$	$.83$	$-.5$	$-.54$

(a) $f(-2)$ undefined

(b) $\lim\limits_{x\to -2} f(x)$ dne $\lim\limits_{x\to 2^-} f(x) \neq \lim\limits_{x\to 2^+} f(x)$

(c) $f(0) = 4$

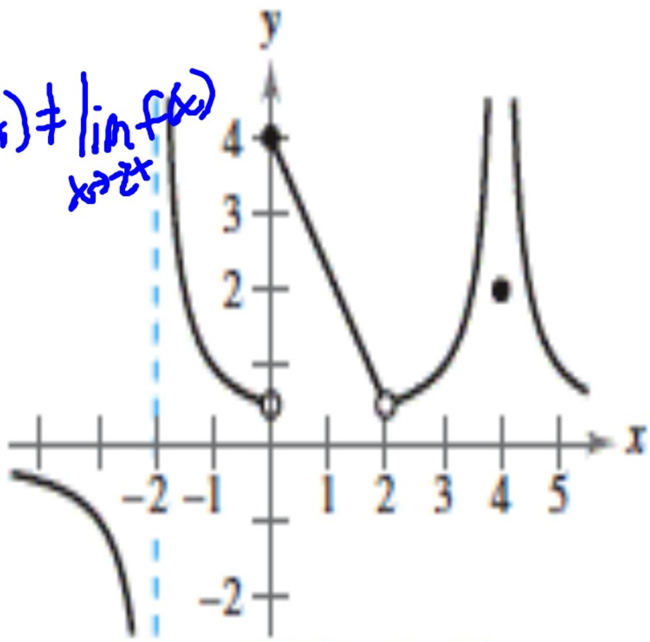
(d) $\lim\limits_{x\to 0} f(x)$ dne

(e) $f(2)$ undefined

(f) $\lim\limits_{x\to 2} f(x) = \frac{1}{2}$

(g) $f(4) = 2$

(h) $\lim\limits_{x\to 4} f(x)$ ∞ or dne



p. 55 #26

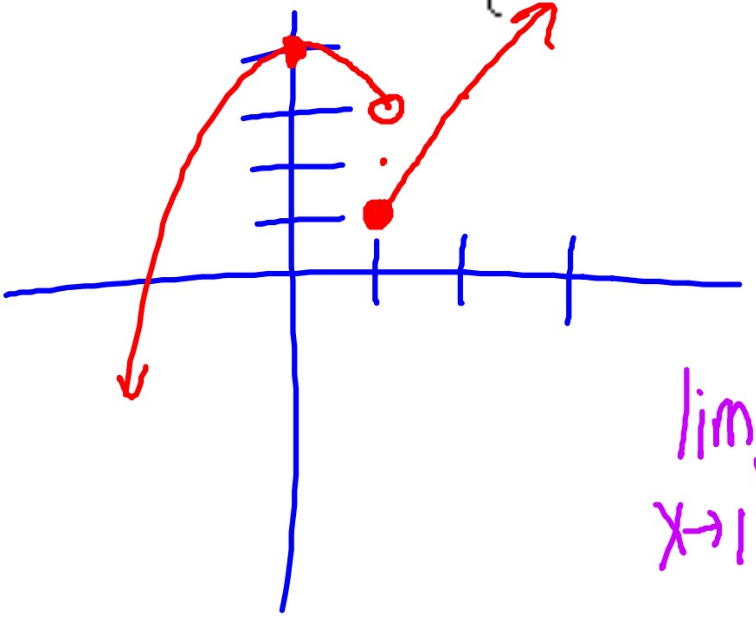
Sketching Piecewise functions

Sketch the graph of the following piecewise function.

$$g(x) = \begin{cases} -x^2+4 & \text{if } x<1 \\ 2x-1 & \text{if } x \geq 1 \end{cases}$$

(1,3) open

(1, 1) closed



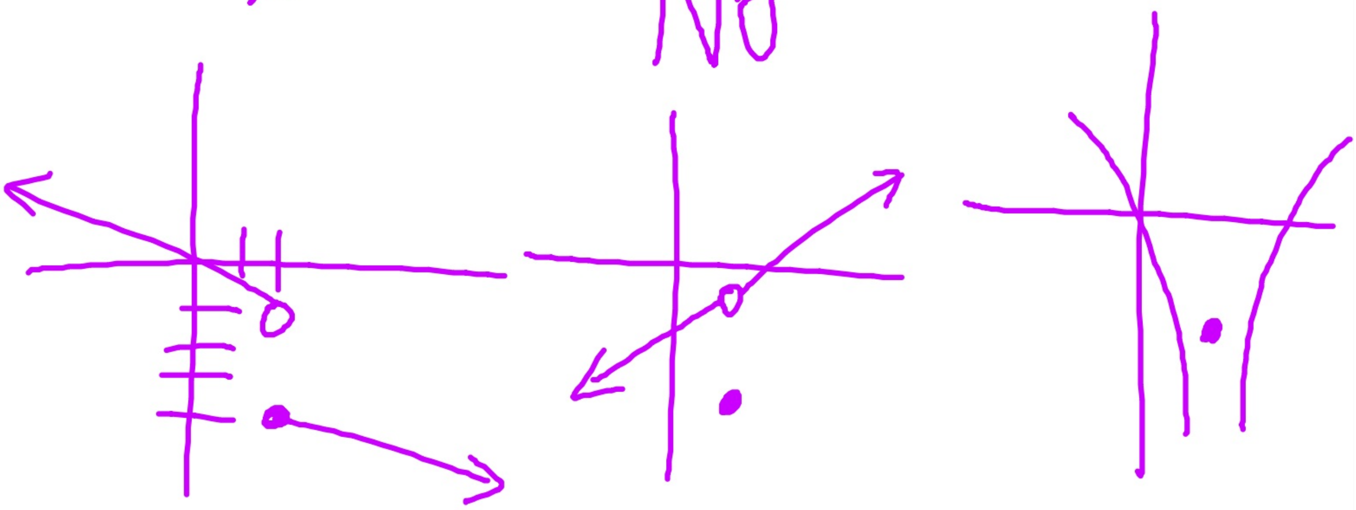
$\lim_{x\to 1} g(x)$ dne

$\lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x)$

If $f(2) = -4$, does that imply that

$$\lim_{x \to 2} f(x) = -4\ ?$$

No



If $\lim_{x \to 2} f(x) = -4$, can you conclude that $f(2) = -4$?

No