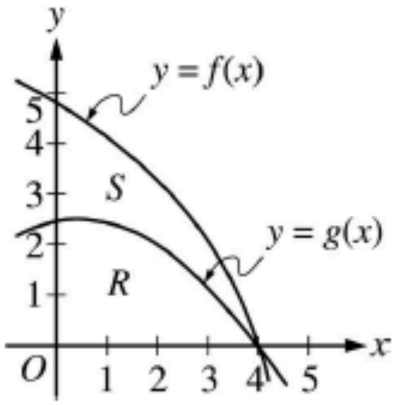
AP Calculus BC Name________________________

HW #4 Free Response Period________Date________________

#1: (Calculator)



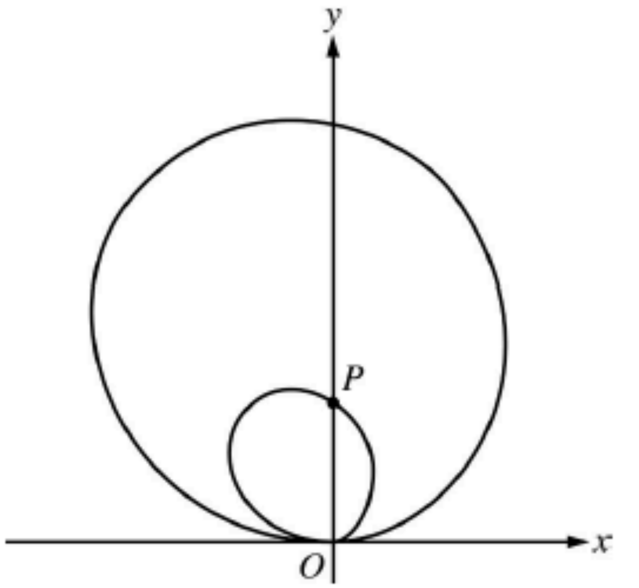
. Let R be the region in the first quadrant bounded by the graph of g, and let S be the region in the first quadrant between the graphs of f and g, as shown in the figure above. The region in the first quadrant bounded by the graph of f and the coordinate axes has area 12.142. The function g is given by $g(x) = \left(\sqrt{x+6}\right)\cos\left(\frac{\pi x}{8}\right)$, and the function f is not explicitly given. The graphs of f and g intersect at the point $(4, 0)$.

(A) Find the area of S.

(B) A solid is generated when S is revolved about the horizontal line $y = 5$. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(C) Region R is the base of an art sculpture. At all points in R at a distance x from the y-axis, the height of the sculpture is given by $h(x) = 4 - x$. Find the volume of the art sculpture.

#2: (Calculator)



Let r be the function given by $r(\theta) = 3\theta \sin\theta$ for $0 \le \theta \le 2\pi$. The graph of r in polar coordinates consists of two loops, as shown in the figure above. Point P is on the graph of r and the y-axis.

(A) Find the rate of change of the x-coordinate with respect to θ at the point P.

(B) Find the area of the region between the inner and outer loops of the graph.

(C) The function r satisfies $\frac{dr}{d\theta} = 3\sin\theta + 3\theta\cos\theta$. For $0 \le \theta \le 2\pi$, find the value of θ that gives the point on the graph that is farthest from the origin. Justify your answer.

#3: (No calculator)

t (minutes)	0	3	5	6	9
$r(t)$ (rotations per minute)	72	95	112	77	50

Rochelle rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Rochelle's ride is modeled by a differentiable function r for $0 \le t \le 9$ minutes. Values of $r(t)$ for selected values of t are shown in the table above.

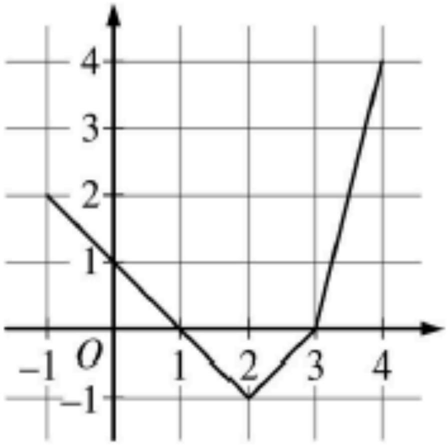
(A) Estimate $r'(4)$. Show the computations that lead to your answer. Indicate units of measure.

(B) Is there a time t, for $3 \le t \le 5$, at which $r(t)$ is 106 rotations per minute? Justify your answer.

(C) Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^9 r(t)\,dt$. Using correct units, explain the meaning of $\int_0^9 r(t)\,dt$ in the context of the problem.

(D) Sarah also rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Sarah's ride is modeled by the function s, defined by $s(t) = 40 + 20\pi \sin\left(\frac{\pi t}{18}\right)$ for $0 \le t \le 9$ minutes. Find the average number of rotations per minute of the wheel of the stationary bicycle for $0 \le t \le 9$ minutes.

#4(No calculator)



Graph of f

Let f be a continuous function defined on the closed interval $-1 \le x \le 4$. The graph of f, consisting of three line segments, is shown above. Let g be the function defined by $g(x) = 5 + \int_2^x f(t)\,dt$ for $-1 \le x \le 4$.

(A) Find $g(4)$.

(B) On what intervals is g increasing? Justify your answer.

(C) On the closed interval $-1 \le x \le 4$, find the absolute minimum value of g and find the absolute maximum value of g. Justify your answers.

(D) Let $h(x) = x \cdot g(x)$. Find $h'(2)$.

#5: (No calculator)

Consider the function f given by $f(x) = xe^{-2x}$ for all $x \geq 0$.

(A) Find $\lim_{x \to \infty} f(x)$.

(B) Find the maximum value of f for $x \geq 0$. Justify your answer.

(C) Evaluate $\int_0^{\infty} f(x)\, dx$, or show that the integral diverges.

#6: (No calculator)

The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n(n+1)} = 1 + \frac{x-2}{3 \cdot 2} + \frac{(x-2)^2}{3^2 \cdot 3} + \frac{(x-2)^3}{3^3 \cdot 4} + \cdots + \frac{(x-2)^n}{3^n(n+1)} + \cdots$$

for all real numbers x for which the series converges.

(A) Determine the interval of convergence of the power series for f. Show the work that leads to your answer.

(B) Find the value of $f''(2)$.

(C) Use the first three nonzero terms of the power series for f to approximate $f(1)$. Use the alternating series error bound to show that this approximation differs from $f(1)$ by less than $\frac{1}{100}$.