

AP Calculus BC  
Homework #2

Name \_\_\_\_\_  
Period \_\_\_\_\_ Date \_\_\_\_\_

**\*You may print out the questions or use your own paper**

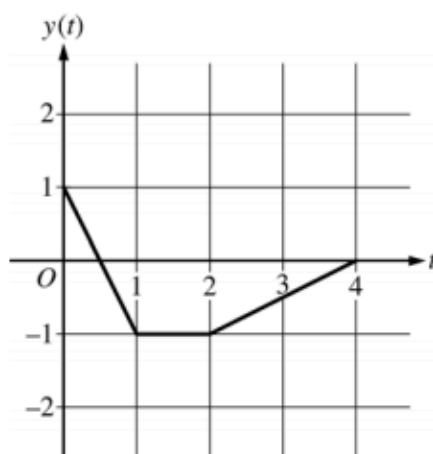
1) (Calculator)

$t$ (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by  $W(t) = 2000e^{-t^2/20}$  liters per hour for  $0 \leq t \leq 8$ , where  $t$  is measured in hours. Water is removed from the tank at a rate modeled by  $R(t)$  liters per hour, where  $R$  is differentiable and decreasing on  $0 \leq t \leq 8$ . Selected values of  $R(t)$  are shown in the table above. At time  $t = 0$ , there are 50,000 liters of water in the tank.

- (a) Estimate  $R'(2)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For  $0 \leq t \leq 8$ , is there a time  $t$  when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

2) (Calculator)



At time  $t$ , the position of a particle moving in the  $xy$ -plane is given by the parametric functions  $(x(t), y(t))$ , where  $\frac{dx}{dt} = t^2 + \sin(3t^2)$ . The graph of  $y$ , consisting of three line segments, is shown in the figure above.

At  $t = 0$ , the particle is at position  $(5, 1)$ .

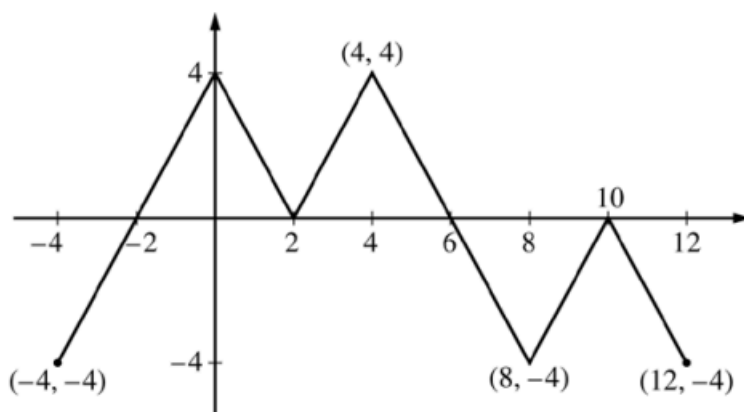
- (a) Find the position of the particle at  $t = 3$ .
- (b) Find the slope of the line tangent to the path of the particle at  $t = 3$ .
- (c) Find the speed of the particle at  $t = 3$ .
- (d) Find the total distance traveled by the particle from  $t = 0$  to  $t = 2$ .

3) (No calculator)

The figure above shows the graph of the piecewise-linear function  $f$ . For  $-4 \leq x \leq 12$ , the function  $g$  is defined by

$$g(x) = \int_2^x f(t) dt.$$

- Does  $g$  have a relative minimum, a relative maximum, or neither at  $x = 10$ ? Justify your answer.
- Does the graph of  $g$  have a point of inflection at  $x = 4$ ? Justify your answer.
- Find the absolute minimum value and the absolute maximum value of  $g$  on the interval  $-4 \leq x \leq 12$ . Justify your answers.
- For  $-4 \leq x \leq 12$ , find all intervals for which  $g(x) \leq 0$ .



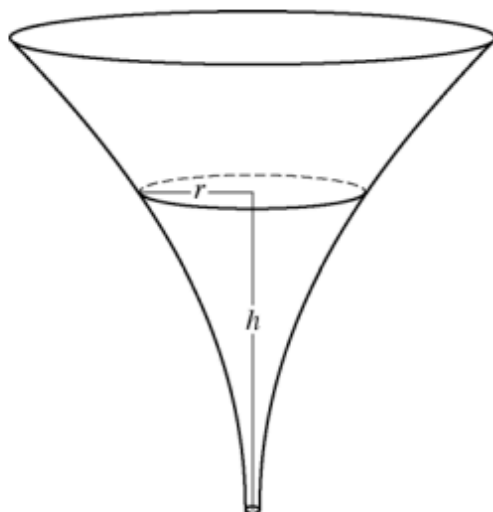
Graph of  $f$

4) (No calculator)

Consider the differential equation  $\frac{dy}{dx} = x^2 - \frac{1}{2}y$ .

- (a) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .
- (b) Let  $y = f(x)$  be the particular solution to the given differential equation whose graph passes through the point  $(-2, 8)$ . Does the graph of  $f$  have a relative minimum, a relative maximum, or neither at the point  $(-2, 8)$ ? Justify your answer.
- (c) Let  $y = g(x)$  be the particular solution to the given differential equation with  $g(-1) = 2$ . Find  $\lim_{x \rightarrow -1} \left( \frac{g(x) - 2}{3(x+1)^2} \right)$ . Show the work that leads to your answer.
- (d) Let  $y = h(x)$  be the particular solution to the given differential equation with  $h(0) = 2$ . Use Euler's method, starting at  $x = 0$  with two steps of equal size, to approximate  $h(1)$ .

5) (No calculator)



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height  $h$ , the radius of the funnel is given by  $r = \frac{1}{20}(3 + h^2)$ , where  $0 \leq h \leq 10$ . The units of  $r$  and  $h$  are inches.

- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is  $h = 3$  inches, the radius of the surface of the liquid is decreasing at a rate of  $\frac{1}{5}$  inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

6) (No calculator)

The function  $f$  has a Taylor series about  $x = 1$  that converges to  $f(x)$  for all  $x$  in the interval of convergence.

It is known that  $f(1) = 1$ ,  $f'(1) = -\frac{1}{2}$ , and the  $n$ th derivative of  $f$  at  $x = 1$  is given by

$$f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n} \text{ for } n \geq 2.$$

- (a) Write the first four nonzero terms and the general term of the Taylor series for  $f$  about  $x = 1$ .
- (b) The Taylor series for  $f$  about  $x = 1$  has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.
- (c) The Taylor series for  $f$  about  $x = 1$  can be used to represent  $f(1.2)$  as an alternating series. Use the first three nonzero terms of the alternating series to approximate  $f(1.2)$ .
- (d) Show that the approximation found in part (c) is within 0.001 of the exact value of  $f(1.2)$ .