AB/BC MC*

The questions with a asterisk should be finished in under one minute

*1)

Which of the following integral expressions is equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{1+\frac{3k}{n}}\cdot\frac{1}{n}\right)$?

(A) $\int_0^1 \sqrt{1+3x}\,dx$

(B) $\int_0^3 \sqrt{1+x}\,dx$

(C) $\int_1^4 \sqrt{x}\,dx$

(D) $\frac{1}{3}\int_0^3 \sqrt{x}\,dx$

*2)

$\int e^x \cos\left(e^x+1\right)dx =$

(A) $\sin\left(e^x+1\right)+C$

(B) $e^x\sin\left(e^x+1\right)+C$

(C) $e^x\sin\left(e^x+x\right)+C$

(D) $\frac{1}{2}\cos^2\left(e^x+1\right)+C$

3)

The area of the region enclosed by the graph of $y=x^2+1$ and the line $y=5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

4)

$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

*5)

Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

*6)

x	0	1	2
$f(x)$	5	2	-7
$f'(x)$	-2	-5	-14

The table above gives selected values of a differentiable and decreasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(2)$?

(A) $-\frac{1}{5}$

(B) $-\frac{1}{14}$

(C) $\frac{1}{5}$

(D) 5

*7)

A curve is defined by the parametric equations $x(t) = 3e^{2t}$ and $y(t) = e^{3t} - 1$.

What is $\frac{d^2y}{dx^2}$ in terms of t ?

(A) $\frac{1}{12e^t}$

(B) $\frac{1}{9e^t}$

(C) $\frac{e^t}{2}$

(D) $\frac{3e^t}{4}$

8)

The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

*9)

. $\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent

10)

$\int_0^1 \sqrt{x}\,(x+1)\,dx =$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

11)

$\int_1^\infty \frac{x}{(1+x^2)^2}\,dx$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

*12)

What is the value of $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$?

(A) $-\frac{15}{8}$ (B) $-\frac{9}{8}$ (C) $-\frac{3}{8}$ (D) $\frac{9}{8}$ (E) $\frac{15}{8}$

13)

Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta$ (B) $3\int_0^{\pi} \cos^2\theta\,d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta\,d\theta$ (E) $3\int_0^{\pi} \cos\theta\,d\theta$

14)

$$\int_1^e \left(\frac{x^2-1}{x}\right)dx =$$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

*15)

If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x=$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

*16)

If $f(x)=\tan(2x)$, then $f'\left(\frac{\pi}{6}\right)=$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

17)

The base of a solid S is the region enclosed by the graph of $y=\sqrt{\ln x}$, the line $x=e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3-1)$

*18)

In the xy-plane, the graph of the parametric equations $x=5t+2$ and $y=3t$, for $-3\le t\le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

*19)

If $\frac{dy}{dx}=\sqrt{1-y^2}$, then $\frac{d^2y}{dx^2}=$

(A) $-2y$ (B) $-y$ (C) $\frac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\frac{1}{2}$

*20)

The length of the path described by the parametric equations $x=\frac{1}{3}t^3$ and $y=\frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2+1}\,dt$

(B) $\int_0^1 \sqrt{t^2+t}\,dt$

(C) $\int_0^1 \sqrt{t^4+t^2}\,dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4+t^4}\,dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2+9}\,dt$

*21)

$\int \frac{5}{1+x^2}dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2}+C$

(B) $\frac{5}{2x}\ln\left(1+x^2\right)+C$

(C) $5x-\frac{5}{x}+C$

(D) $5\arctan x+C$

(E) $5\ln\left(1+x^2\right)+C$

22)

The region in the first quadrant bounded by the graph of $y=\sec x$, $x=\frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$

(B) $\pi-1$

(C) π

(D) 2π

(E) $\frac{8\pi}{3}$

*23)

$\int \tan(2x)\,dx =$

(A) $-2\ln\left|\cos(2x)\right|+C$

(B) $-\frac{1}{2}\ln\left|\cos(2x)\right|+C$

(C) $\frac{1}{2}\ln\left|\cos(2x)\right|+C$

(D) $2\ln\left|\cos(2x)\right|+C$

(E) $\frac{1}{2}\sec(2x)\tan(2x)+C$

24)

A series expansion of $\frac{\sin t}{t}$ is

(A) $1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

(B) $\frac{1}{t}-\frac{t}{2!}+\frac{t^3}{4!}-\frac{t^5}{6!}+\cdots$

(C) $1+\frac{t^2}{3!}+\frac{t^4}{5!}+\frac{t^6}{7!}+\cdots$

(D) $\frac{1}{t}+\frac{t}{2!}+\frac{t^3}{4!}+\frac{t^5}{6!}+\cdots$

(E) $t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots$

25)

$\int \arcsin x \, dx =$

(A) $\sin x-\int \frac{x\,dx}{\sqrt{1-x^2}}$

(B) $\frac{(\arcsin x)^2}{2}+C$

(C) $\arcsin x+\int \frac{dx}{\sqrt{1-x^2}}$

(D) $x\arccos x-\int \frac{x\,dx}{\sqrt{1-x^2}}$

(E) $x\arcsin x-\int \frac{x\,dx}{\sqrt{1-x^2}}$

26)

x	0	2	4	8
$f(x)$	3	4	9	13
$f'(x)$	0	1	1	2

The table above gives values of a differentiable function f and its derivative at selected values of x. If h is the function given by $h(x) = f(2x)$, which of the following statements must be true?

(I) h is increasing on $2 < x < 4$.

(II) There exists c, where $0 < c < 4$, such that $h(c) = 12$.

(III) There exists c, where $0 < c < 2$, such that $h'(c) = 3$.

(A) II only

(B) I and III only

(C) II and III only

(D) I, II, and III

27)

. Let f be a function with second derivative $f''(x) = \sqrt{1 + 3x}$. The coefficient of x^3 in the Taylor series for f about $x = 0$ is

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{3}{2}$

*28)

. If $F(x) = \int_4^{x^2} \sqrt{t}\, dt$ for all real numbers $x > 0$, then $F'(x) =$

(A) $-\frac{1}{2x}$ (B) $\sqrt{x}$ (C) x (D) $2x^2$ (E) $\frac{2x^3 - 16}{3}$

*29)

If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

x	$f(x)$
-1	4
0	5
1	7

(E)

x	$f(x)$
-1	4
0	6
1	7

30)

. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

(A) $\frac{1}{1440}$ (B) $\frac{81}{160}$ (C) $\frac{9}{4}$ (D) $\frac{9}{2}$ (E) $\frac{27}{2}$

1 a
2 a
3 d
4 c
5 c
6 a
7 a
8 a
9 b
10 c
11 c
12 d
13 a
14 e
15 c
16 e
17 c
18 a
19 b
20 c
21 d
22 c
23 b
24 a
25 e
26 c
27 c
28 d
29 e
30 c