AP Calculus
Rate in/Rate out

1)

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a)   How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b)   The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c)   Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d)   At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?
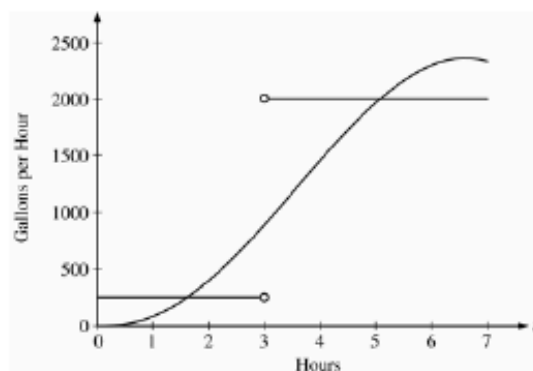
2)

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is
$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

Answers

(a) $\int_{9}^{17} E(t)\,dt = 6004.270$

6004 people entered the park by 5 pm.

$3 \begin{cases} 1: \text{ limits} \\ 1: \text{ integrand} \\ 1: \text{ answer} \end{cases}$

(b) $15\int_{9}^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was $104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was

$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104{,}041$.

$1: \text{ setup}$

(c) $H'(17) = E(17) - L(17) = -380.281$

There were 3725 people in the park at $t = 17$.
The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

$3 \begin{cases} 1: \text{ value of } H'(17) \\ 2: \text{ meanings} \\ \qquad 1: \text{ meaning of } H(17) \\ \qquad 1: \text{ meaning of } H'(17) \\ \qquad <-1> \text{ if no reference to } t = 17 \end{cases}$

(d) $H'(t) = E(t) - L(t) = 0$

$t = 15.794 \text{ or } 15.795$

$2 \begin{cases} 1: E(t) - L(t) = 0 \\ 1: \text{ answer} \end{cases}$

1)

2)

(a) $\int_{0}^{7} f(t)\,dt \approx 8264$ gallons

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ for $0 \le t < 1.617$ and $3 < t < 5.076$.

$2: \begin{cases} 1: \text{intervals} \\ 1: \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0$, 3, and 7.

$5: \begin{cases} 1: \text{identifies } t = 3 \text{ as a candidate} \\ 1: \text{integrand} \\ 1: \text{amount of water at } t = 3 \\ 1: \text{amount of water at } t = 7 \\ 1: \text{conclusion} \end{cases}$

t (hours)	gallons of water
0	5000
3	$5000 + \int_{0}^{3} f(t)\,dt - 250(3) = 5126.591$
7	$5126.591 + \int_{3}^{7} f(t)\,dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.