AP Calculus - Chapter 6/7 Review

1.

At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

2.

If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

3.

The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

4.

If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3} + e^{-x}$ (C) $1 + e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

5.

If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

6.

If $\frac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

7.

If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

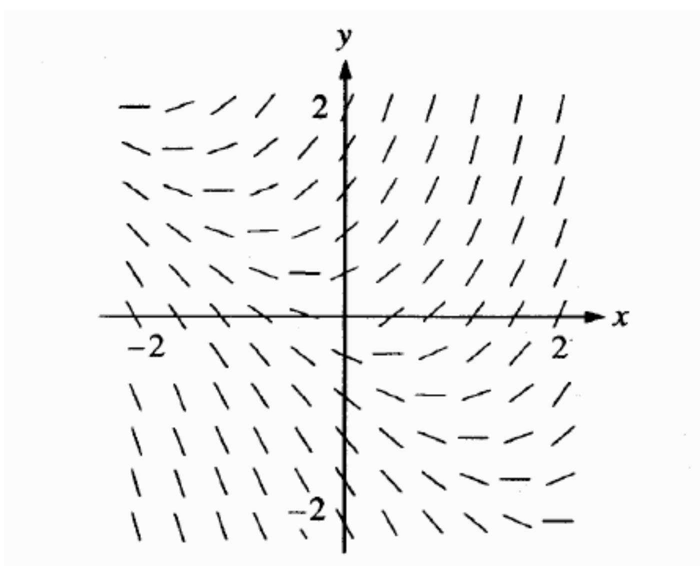
8.

If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$

(D) $\tan x + 5$ (E) $\tan x + 5e^x$

9.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

10.

The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{(1-e^{-6})}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1-e^{-3}$

11.

The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{4\pi}{3}$ (B) $\frac{16\pi}{5}$ (C) $\frac{4}{3}$ (D) $\frac{16}{5}$ (E) $\frac{64}{5}$

12.

The area of the region in the first quadrant enclosed by the graph of $y = x(1-x)$ and the x-axis is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) 1

13.

The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

14.

The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when R is revolved about the y-axis is

(A) 6π (B) 8π (C) $\frac{34\pi}{3}$ (D) 16π (E) $\frac{544\pi}{15}$

15.

The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

16.

What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \frac{\pi}{3}$?

(A) $\frac{\pi}{\sqrt{3}}$

(B) π

(C) $\pi\sqrt{3}$

(D) $\frac{8\pi}{3}$

(E) $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$

17.

The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\,\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

18.

The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

19.

The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

20.

The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

(A) 36 (B) 23 (C) 20 (D) 17 (E) 9

21.

The area of the region in the first quadrant that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$

22.

The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

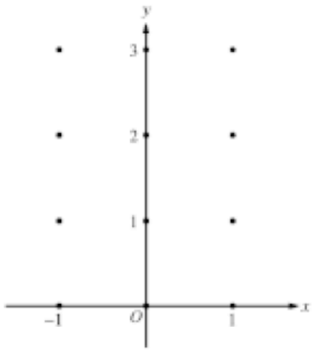
23.

The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e - 1$ (C) $2(e-1)$ (D) $2e - 1$ (E) $2e$

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$ (no calculator)

a. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

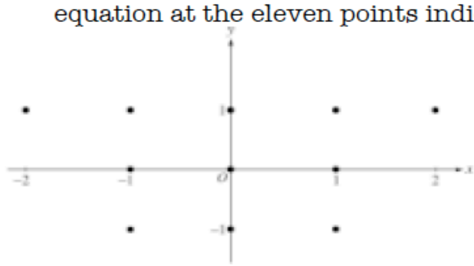


b. Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = 3.

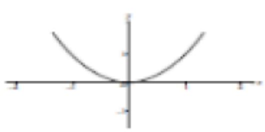
c. Write an equation of the line tangent to y = f(x) at x = 2.

2. Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$ (no calculator)

a. On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.



b. Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



c. Find the particular solution y = f(x) to the given differential equation with the initial condition f(0) = -1

d. Find the range of the solution found in part (c).

3. (no calculator)

. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

4. (calculator)

. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2 + 1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.

(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find $f(1.2)$.

5. (calculator)

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

6. (no calculator)

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

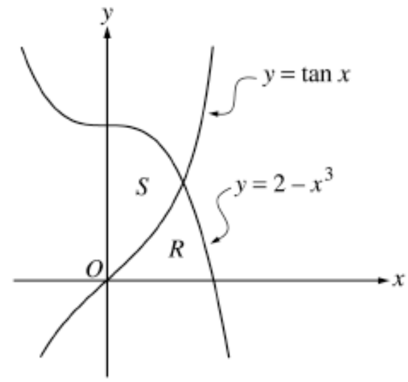
(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

7. (calculator)

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



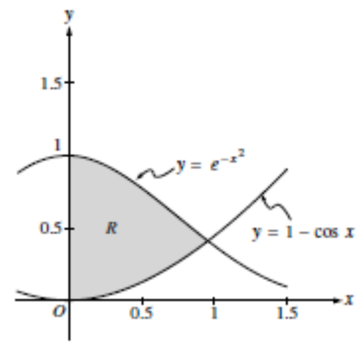
(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

8. (calculator)

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.



(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

9. (no calculator)

Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

(a) Find the area of the region R.

(b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

(c) Find the volume of the solid generated when R is revolved about the x–axis.

(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

Answers

Multiple choice
1 A
2 C
3 C
4 D
5 B
6 C
7 B
8 C
9 C
10 A
11 D
12 A
13 C
14 B
15 D
16 C
17 C
18 C
19 C
20 D
21 A
22 A
23 C

Free response
1b $y = 3e^{x^2/4}$
1c y – 3e = 3e(x – 2)

2b There is a horizontal asymptote at y = 1
2c $y = \frac{-2}{x^2+1} + 1$
2d range: [-1, 1)

3a -1/8
3b $y = \frac{1}{x^2 - 6x + 13}$

4a ½
4b 4.1
4c $f(x) = \sqrt{x^3 + x + 14}$
4d 4.114

5a 6004 people entered the park by 5 pm
5b The amount collected was $104,048
5c There were 375 people in the park at t=17. Since $H'(17) = E(17) - L(17) = -380.281 < 0$, the number of people in the park was decreasing at a rate of approximately 380 people/hr at t=17 hrs.
5d t=15.794 hrs by the Extreme Value Theorem

6a 14/3
6b 148/3
6c $A(t) = 30 + \int_0^t (8 - \sqrt{x+1})\,dx$ or

$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$
6d t = 63

7a 0.729
7b 1.160 or 1.161
7c 8.331 or 8.332

8a 0.590 or 0.591
8b 1.746 or 1.747
8c 0.461

9a 16/3
9b $4^{2/3}$
9c 8π
9d $2\sqrt{2}$