

AP Chapter 4 Review

Multiple Choice

1.

The average value of \sqrt{x} over the interval $0 \leq x \leq 2$ is

- (A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

2.

If $F(x) = \int_0^x e^{-t^2} dt$, then $F'(x) =$

- (A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\frac{e^{-x^2+1}}{-x^2+1} - e$
 (D) $e^{-x^2} - 1$ (E) e^{-x^2}

3.

If $F(x) = \int_1^{x^2} \sqrt{1+t^3} dt$, then $F'(x) =$

- (A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$
 (D) $\sqrt{1+x^3}$ (E) $\int_1^{x^2} \frac{3t^2}{2\sqrt{1+t^3}} dt$

4.

If $\int_{-2}^2 (x^7 + k) dx = 16$, then $k =$

- (A) -12 (B) -4 (C) 0 (D) 4 (E) 12

5.

If $\int_a^b f(x) dx = a + 2b$, then $\int_a^b (f(x) + 5) dx =$

- (A) $a + 2b + 5$ (B) $5b - 5a$ (C) $7b - 4a$ (D) $7b - 5a$ (E) $7b - 6a$

6.

A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t - 3t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

- (A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

7.

$$\int_0^{\pi/4} \tan^2 x \, dx =$$

- (A) $\frac{\pi}{4} - 1$ (B) $1 - \frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2} - 1$ (E) $\frac{\pi}{4} + 1$

8.

$$\int x \sqrt{4-x^2} \, dx =$$

- (A) $\frac{(4-x^2)^{3/2}}{3} + C$ (B) $-(4-x^2)^{3/2} + C$ (C) $\frac{x^2(4-x^2)^{3/2}}{3} + C$
 (D) $-\frac{x^2(4-x^2)^{3/2}}{3} + C$ (E) $-\frac{(4-x^2)^{3/2}}{3} + C$

9.

$$\int_0^3 |x-1| \, dx =$$

- (A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

10.

If $\int_0^k (2kx - x^2) \, dx = 18$, then $k =$

- (A) -9 (B) -3 (C) 3 (D) 9 (E) 18

11.

The average value of $f(x) = x^2 \sqrt{x^3 + 1}$ on the closed interval $[0, 2]$ is

- (A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26

12.

$$\int \sin(2x+3) \, dx =$$

- (A) $-2 \cos(2x+3) + C$ (B) $-\cos(2x+3) + C$ (C) $-\frac{1}{2} \cos(2x+3) + C$
 (D) $\frac{1}{2} \cos(2x+3) + C$ (E) $\cos(2x+3) + C$

13.

If the velocity of a particle moving along the x -axis is $v(t) = 2t - 4$ and if at $t = 0$ its position is 4, then at any time t its position $x(t)$ is

- (A) $t^2 - 4t$ (B) $t^2 - 4t - 4$ (C) $t^2 - 4t + 4$ (D) $2t^2 - 4t$ (E) $2t^2 - 4t + 4$

14.

If F and f are continuous functions such that $F'(x) = f(x)$ for all x , then $\int_a^b f(x) dx$ is

- (A) $F'(a) - F'(b)$
 (B) $F'(b) - F'(a)$
 (C) $F(a) - F(b)$
 (D) $F(b) - F(a)$
 (E) none of the above

15.

$$\int_0^{\frac{\pi}{3}} \sin(3x) dx =$$

- (A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

16.

If $\int_1^{10} f(x) dx = 4$ and $\int_{10}^3 f(x) dx = 7$, then $\int_1^3 f(x) dx =$

- (A) -3 (B) 0 (C) 3 (D) 10 (E) 11

17.

$$\int_0^2 \sqrt{4-x^2} dx =$$

- (A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

18.

$$\int \frac{x \, dx}{\sqrt{3x^2 + 5}} =$$

- (A) $\frac{1}{9}(3x^2 + 5)^{\frac{3}{2}} + C$
- (B) $\frac{1}{4}(3x^2 + 5)^{\frac{3}{2}} + C$
- (C) $\frac{1}{12}(3x^2 + 5)^{\frac{1}{2}} + C$
- (D) $\frac{1}{3}(3x^2 + 5)^{\frac{1}{2}} + C$
- (E) $\frac{3}{2}(3x^2 + 5)^{\frac{1}{2}} + C$

19.

The acceleration of a particle moving along the x -axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

- (A) $9t^2 + 1$
- (B) $3t^2 - 2t + 4$
- (C) $t^3 - t^2 + 4t + 6$
- (D) $t^3 - t^2 + 9t - 20$
- (E) $36t^3 - 4t^2 - 77t + 55$

20.

If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x , which of the following is NOT necessarily true?

- (A) $\int_{-1}^1 f(x) \, dx > 0$
- (B) $\int_{-1}^1 2f(x) \, dx = 2 \int_{-1}^1 f(x) \, dx$
- (C) $\int_{-1}^1 f(x) \, dx = 2 \int_0^1 f(x) \, dx$
- (D) $\int_{-1}^1 f(x) \, dx = - \int_1^{-1} f(x) \, dx$
- (E) $\int_{-1}^1 f(x) \, dx = \int_{-1}^0 f(x) \, dx + \int_0^1 f(x) \, dx$

21.

$$\int_0^1 x^3 e^{x^4} dx =$$

- (A) $\frac{1}{4}(e-1)$ (B) $\frac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

22.

$$\int \tan(2x) dx =$$

- (A) $-2 \ln |\cos(2x)| + C$ (B) $-\frac{1}{2} \ln |\cos(2x)| + C$ (C) $\frac{1}{2} \ln |\cos(2x)| + C$
 (D) $2 \ln |\cos(2x)| + C$ (E) $\frac{1}{2} \sec(2x) \tan(2x) + C$

23.

The average value of $\frac{1}{x}$ on the closed interval $[1, 3]$ is

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\ln 3}{2}$ (E) $\ln 3$

24.

Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}} dx$?

- (A) $\arcsin \frac{x}{5} + C$ (B) $\arcsin x + C$ (C) $\frac{1}{5} \arcsin \frac{x}{5} + C$
 (D) $\sqrt{25-x^2} + C$ (E) $2\sqrt{25-x^2} + C$

25.

$$\frac{1}{2} \int e^{\frac{t}{2}} dt =$$

- (A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^t + C$

26.

If $\int_1^2 f(x-c)dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)dx =$

- (A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) -5

27.

$$\int_2^3 \frac{x}{x^2+1} dx =$$

- (A) $\frac{1}{2} \ln \frac{3}{2}$ (B) $\frac{1}{2} \ln 2$ (C) $\ln 2$ (D) $2 \ln 2$ (E) $\frac{1}{2} \ln 5$

28.

For all $x > 1$, if $f(x) = \int_1^x \frac{1}{t} dt$, then $f'(x) =$

- (A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x

29.

$$\int \frac{5}{1+x^2} dx =$$

- (A) $\frac{-10x}{(1+x^2)^2} + C$ (B) $\frac{5}{2x} \ln(1+x^2) + C$ (C) $5x - \frac{5}{x} + C$
(D) $5 \arctan x + C$ (E) $5 \ln(1+x^2) + C$

30.

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$$

- (A) $\ln \sqrt{2}$ (B) $\ln \frac{\pi}{4}$ (C) $\ln \sqrt{3}$ (D) $\ln \frac{\sqrt{3}}{2}$ (E) $\ln e$

31.

$$\int_1^e \left(\frac{x^2 - 1}{x} \right) dx =$$

- (A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

32.

$$\int_0^1 \sqrt{x^2 - 2x + 1} dx \text{ is}$$

- (A) -1
 (B) $-\frac{1}{2}$
 (C) $\frac{1}{2}$
 (D) 1
 (E) none of the above

33.

Choose the integral expression that would result in the total distance traveled on the interval $[0, 3]$ if the velocity is given by $v(t) = e^t - 6$.

- (A) $\int_0^{\ln 6} (e^t - 6) dt + \int_{\ln 6}^3 (e^t - 6) dt$
 (B) $\int_3^{\ln 6} (e^t - 6) dt - \int_{\ln 6}^0 (e^t - 6) dt$
 (C) $\int_{\ln 6}^3 (e^t - 6) dt - \int_0^{\ln 6} (e^t - 6) dt$
 (D) $\int_0^{\ln 6} (e^t - 6) dt - \int_{\ln 6}^3 (e^t - 6) dt$
 (E) $\int_0^3 (e^t - 6) dt$

34.

When two particles start at the origin with velocities $v(t) = 4 \cos t$ and $v(t) = 4 \sin t$, how many times in the interval $[0, 2\pi]$ will their speeds be equal?

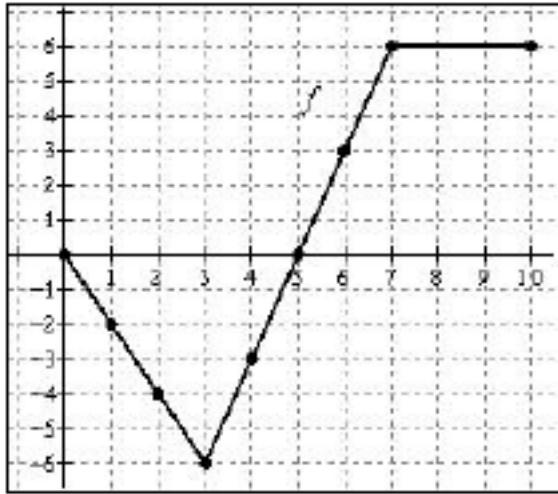
- (A) 2 (B) 3 (C) 4
 (D) 5 (E) 6

Free Response

1.

The graph of the function f shown consists of two line segments. Let g be the function given by

$$g(x) = \int_5^x f(t) dt$$



- a) Find $g(0)$ and $g(10)$
- b) Find $g'(2)$ and $g''(2)$
- c) For what value(s) of x does $g(x)$ have a relative minimum? Justify.
- d) For what value(s) of x does $g(x)$ have a point of inflection? Justify.
- e) For what interval(s) is $g(x)$ increasing? Justify.
- f) For what interval(s) is $g(x)$ concave down? Justify.

2.

Let $v(t)$ be an increasing differentiable function on the interval $[0, 60]$.

t (sec)	0	10	40	60
$v(t)$ (meters per second)	2.0	2.3	2.5	2.6

Approximate $\int_0^{60} v(t) dt$ using the indicated method with 3 sub-intervals.

- a) Left sum; Is this an under or over approximation? Justify.
- b) Right sum; Is this an under or over approximation? Justify.
- c) Trapezoidal sum
- d) Using the trapezoidal approximation and that the initial position was $x(0) = 2$, find $x(60)$.

3.

Water is being pumped into a pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t .

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63

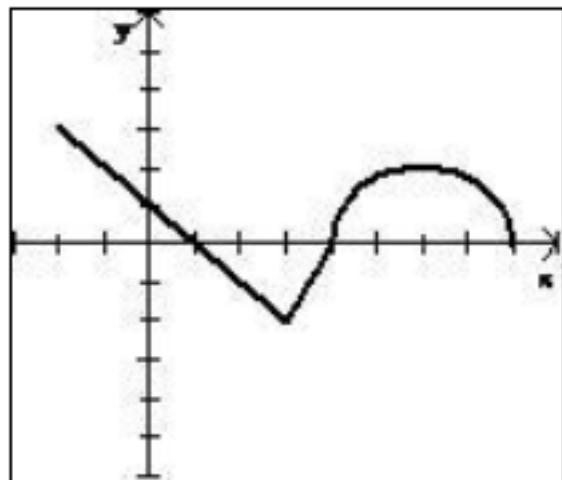
- a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $[0, 12]$.
- b) What does $\frac{1}{12} \int_0^{12} P(t) dt$ represent in the context of this question? Include units of measure with the explanation.

4.

The graph at the right is f' . The graph consists of two line segments and a semicircle. It is given that $f(1) = 5$.

Find

- a) $f(-2)$
b) $f(4)$
c) $f(8)$
d) Find the absolute maximum value on the interval $[-2, 8]$
e) Find the absolute minimum value on the interval $[-2, 8]$



5.

A particle moves along the x -axis with acceleration given by $a(t) = 2t - 10 + \frac{12}{t}$ for $t \geq 1$.

- (a) Write an expression for the velocity $v(t)$, given that $v(1) = 9$.
- (b) For what values of t , $1 \leq t \leq 3$, is the velocity a maximum? Justify your answer.

MC Answers

1 C
2 E
3 A
4 D
5 C
6 C
7 B
8 E
9 D
10 C
11 A
12 C
13 C
14 D
15 D
16 E
17 C
18 D
19 C
20 C
21 A
22 B
23 D
24 A
25 C
26 B
27 B
28 B
29 D
30 A
31 E
32 C
33 C
34 C

Free Response Answers

- 1a 15, 24
1b -4, -2
1c $x = 5$ since $g' = f$ and g is changing from negative to positive at $x = 5$.
1d $x = 3$ because $g' = f$ because the slope of f is changing from negative to positive at $x = 3$
1e g is increasing on the interval $(5, 10)$ since $g' = f$ and $f > 0$ on this interval.
1f g is concave down on the interval $(0, 3)$ since $g' = f$ and f is decreasing on this interval.
- 2a 139 ; under approximation since $v(t)$ is an increasing function
2b 150 ; over approximation since $v(t)$ is an increasing function
2c $289/2$; $x(60)$ is approximately $293/2$
- 3a 660 cubic feet
3b The average rate that water is pumped into the pool from $t = 0$ to $t = 12$.
- 4a $\frac{1}{2}$
4b 2
4c $2 + 2\pi$
4d Check endpoints and relative maximums; the absolute max value is $2 + 2\pi$
4e Check endpoints and relative minimums; the absolute min value is $\frac{1}{2}$.

5.

(a) $v(t) = t^2 - 10t + 12 \ln t + C$

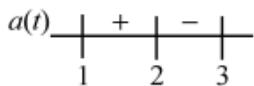
$9 = v(1) = 1 - 10 + 12(0) + C$

$C = 18$

$v(t) = t^2 - 10t + 12 \ln t + 18$

(b) $a(t) = \frac{2t^2 - 10t + 12}{t} = \frac{2(t-2)(t-3)}{t}$

$a(t) = 0$ when $t = 2$ and $t = 3$.



Since the velocity is increasing for $1 \leq t < 2$ and decreasing for $2 < t < 3$, the velocity is a maximum at $t = 2$.