REM 4.11 THE SECOND FUNDAMENTAL THEOREM OF CALCULUS

continuous on an open interval I containing a, then, for every x in the
l,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

FIND F(X) AND F'(X)

#1

. $F(x) = \int_0^x \sin\theta\, d\theta$

$F'(x) = \sin x$

$F(x) = -\cos\theta \Big|_0^x$

$F(x) = -\cos x - (-\cos 0)$

$F(x) = -\cos x + 1$

#2

$$F(x) = \int_0^x (4t - 7)\, dt$$

$$F'(x) = 4x - 7$$

③ $F(x) = \int_1^{x^2} t^3\,dt$

$F(x) = \frac{1}{4}t^4 \Big|_1^{x^2}$

$F(x) = \frac{1}{4}x^8 - \frac{1}{4}\cdot 1$

$F'(x) = 2x^7$

$2x \cdot x^6$

④ $F(\theta) = \int_{1}^{\sin\theta} \sqrt{t^2+6}\, dt$

1. plug in the upper limit
2. Multiply by the derivative of the upper limit

$F'(\theta) = \sqrt{\sin^2\theta + 6} \cdot \cos\theta$

$F'(\pi) = \sqrt{6} \cdot (-1)$

$= -\sqrt{6}$

⑤ $f(x) = \int_1^{x^2} t\sqrt{t+1}$

$f'(x) = x^2\sqrt{x^2+1} \cdot 2x$

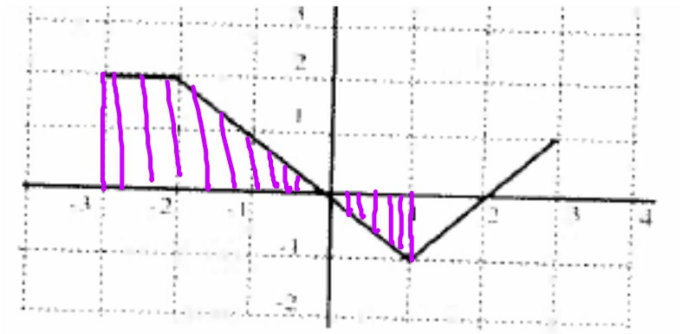
$= 2x^3\sqrt{x^2+1}$

$f'(3) = 54\sqrt{10}$

Accumulation Functions
(2nd Fundamental Theorem)

Let g be defined as $g(x) = \int_{1}^{x} f(t)\,dt$

1) Find g(-3) and g'(-3)



This is the graph of f

$$g(-3) = \int_{1}^{-3} f(t)\,dt$$

$$= -\int_{-3}^{1} f(t)\,dt$$

$$= -\left[4 + -\tfrac{1}{2} \right]$$

$$= -3\tfrac{1}{2}$$

$$g'(x) = f(x)$$

$$g'(-3) = f(-3) = 2$$

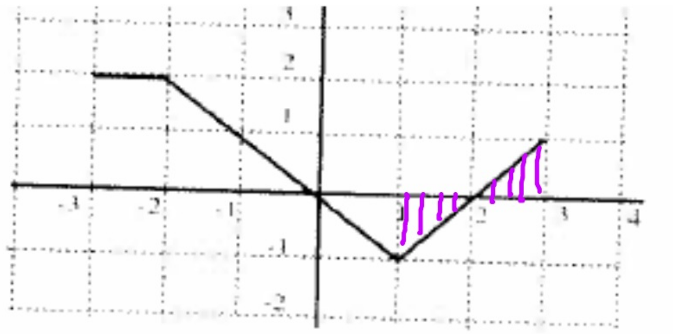
Let g be defined as $g(x) = \int_1^x f(t)\,dt$

2) Find g(3) and g'(3)

$g(3) = \int_1^3 f(t)\,dt$

$= 0$

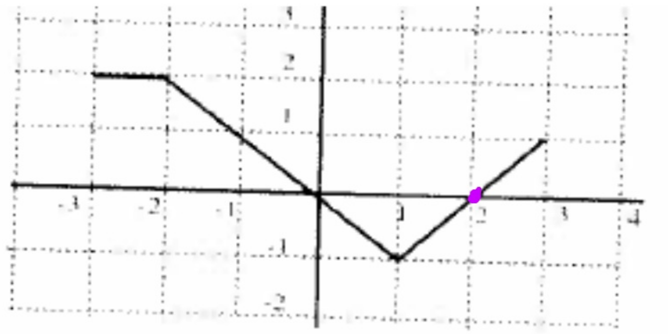
$g'(3) = f(3) = 1$



This is the graph of f

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

3) Find g'(2) and g''(2)



This is the graph of f

$g'(x) = f(x)$

$g'(2) = f(2) = 0$

$g''(x) = f'(x)$

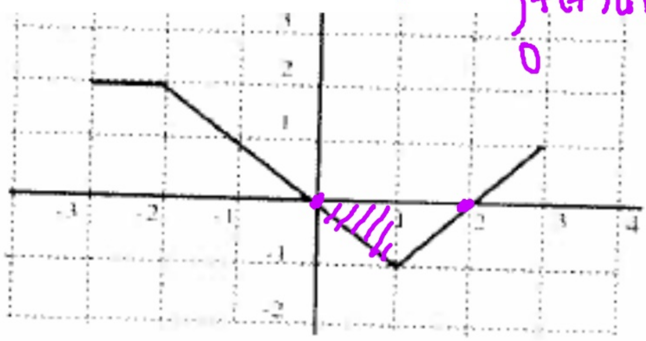
$g''(2) = f'(2) = 1 \leftarrow$ slope at $x=2$

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

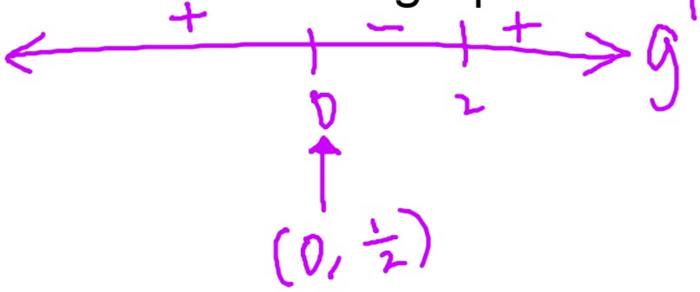
$g(0) = \int_1^0 f(t)\,dt = -\int_0^1 f(t)\,dt$

4) Find all values of x on the open interval (-3, 3) at which g attains a relative maximum. Justify

$g'(x) = f(x)$



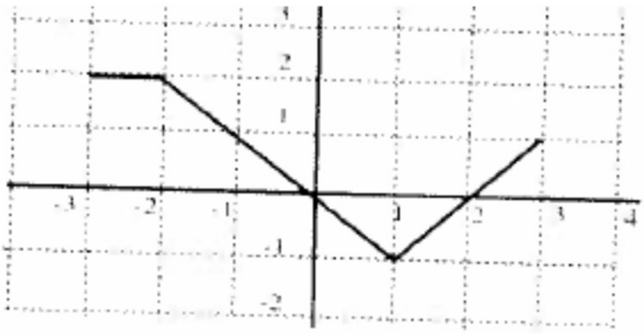
This is the graph of f



Since g'(x) = f(x), there is a relative max at x = 0 because g' changes from positive to negative at this value.

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

5) Find the absolute maximum value of g on the closed interval [-3,3]



This is the graph of f

x	g(x)
-3	$-3\frac{1}{2}$
0	$\frac{1}{2}$
3	0

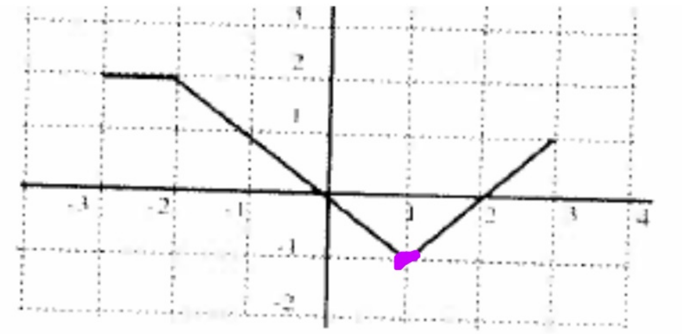
Abs max value is 1/2

check endpoints and critical points (rel max)

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

6) Find all values of x in the open interval (-3,3) at which the graph of g has a point of inflection. Justify. $g'(x) = f(x)$

$x = 1$



This is the graph of f

POI at x = 1 because there is relative extrema for g' at x = 1

because the slope of g' changes signs; because the graph of g' changes from decreasing to increasing