Tabular Data (IVT and MVT)

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

Estimate s'(7). Include units of measure.

(1, 4) (10, 8)

$$s'(7) \approx \frac{8-4}{10-1} = \frac{4}{9} \text{ ft/sec}$$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

Estimate s'(12). Include units of measure.

(10, 8) (12, 3)

$$s'(12) \approx \frac{3-8}{12-10} = \frac{-5}{2} \text{ ft/sec}$$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For 0< t <12, must there be a time t such that s(t) = 6?
How many? 2

(1, 10) (10, 12)

$s(1) < 6 < s(10)$ $s(12) < 6 < s(10)$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For 0< t <12, must there be a time t such that s(t) = 6?

Since s(1)< 6 < s(10) and s(t) is continuous, by IVT there must exist a value c, 1 < c < 10, such that s(c) = 6

AND

since s(12) < 6 < s(10) there must exist a value c, 10 < c < 12, such that s(c) = 6

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For 0< t <12, must there be a time t such that s(t) = 0?

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 12$, must there be a time t such that $s(t) = 0$?

No. s(t) is continuous but there are no intervals (a, b) such that $s(a) < 0 < s(b)$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 12$, must there be a time t such that $s'(t) = 0$?

$$\frac{3-3}{12-0} = 0$$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 12$, must there be a time t such that $s'(t) = 0$?

Yes, since s(t) is continuous and differentiable, by MVT, there exists a value c, $0 < c < 12$, such that $s'(c) = 0$.

(0, 3) (12, 3)

$$\frac{3-3}{12-0} = 0$$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 12$, must there be a time t such that $s'(t) = 0$?

Since s(t) is continuous and differentiable, and $s(0) = s(12)$ by Rolle's theorem there must exist a value c in (0,12) such that $s'(c) = 0$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 10$, must there be a time t such that $s'(t) = 0$?

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For 0 < t < 10, must there be a time t such that s'(t) = 0?

Using Rolle's Theorem as a justification

No, Rolle's Theorem does not apply. s(t) is continuous and differentiable but there are no values on [0,10] such that s(a) = s(b).

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 10$, must there be a time t such that $s'(t) = 0$?

Using MVT as a justification

No.

s(t) is continuous and differentiable,

but on the interval $0 < t < 10$, there are no values a and b such that

$$\frac{s(b) - s(a)}{b - a} = 0$$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 10$, must there be a time t such that $s'(t) = 1/2$?

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 10$, must there be a time t such that $s'(t) = 1/2$?

Yes, since s(t) is continuous and differentiable, by MVT, there exists a value c, $0 < c < 10$, such that $s'(c) = 1/2$

(0, 3) (10, 8)

$$\frac{8-3}{10-0} = \frac{1}{2}$$

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 1$, must there be a time t such that $s'(t) = 2$?

Let s(t) be a differentiable function on [0, 12] where t is in seconds and s(t) is feet

t	0	1	10	12
s(t)	3	4	8	3

For $0 < t < 1$, must there be a time t such that $s'(t) = 2$?

No.

s(t) is continuous and differentiable,

but on the interval $0 < t < 1$,

$\frac{s(1) - s(0)}{1 - 0}$ does not equal 2