

15. Relative Extrema (2nd Derivative Test):

- f has a relative maximum when $f' = 0$ or is **undefined** and $f'' < 0$.
- f has a relative minimum when $f' = 0$ or is **undefined** and $f'' > 0$.

16. Point of Inflection

- f has a point of inflection when f' has relative extrema and f'' changes signs.

17. Fundamental theorem of calculus: $\int_a^b f(x)dx = F(b) - F(a)$

- $\int_a^b f(x)dx$ is the area under the curve of $f(x)$
- $\int_b^a f(x)dx$ is negative if the area is below the x-axis

18. Area Accumulation Functions: $\int_c^{g(x)} f(t)dt$

- To find the derivative: $\frac{d}{dx} \left[\int_c^{g(x)} f(t)dt \right] = f(g(x))g'(x)$ (2ND FTC)

19. Volume by discs (horizontal axis): $\pi \int_a^b r^2 dx$

20. Volume by discs (vertical axis): $\pi \int_a^b r^2 dy$

21. Volume by washers (horizontal axis): $\pi \int_a^b (R^2 - r^2) dx$

22. Volume by washers (vertical axis): $\pi \int_a^b (R^2 - r^2) dy$

23. Volume by cross sections perpendicular to the x-axis: $\int_a^b A(x)dx$

24. Volume by cross sections perpendicular to the y-axis: $\int_a^b A(y)dy$

25. Position/ Velocity/Acceleration (AB):

- Speed is increasing when: acceleration and velocity have the same signs
- Speed is decreasing when: acceleration and velocity have opposite signs

26. Given a graph of f and $g(x) = \int_0^x f(t)dt$:

- The graph f is the graph of g'
- $\int_0^x f(t)dt$ is the AREA under the curve.
- To evaluate $g(x)$, evaluate the integral by using geometric shapes.

27. Derivative Approximations

x	f(x)
a	e
b	f
d	g

To approximate $f'(c) \approx \frac{f(d) - f(b)}{d - b}$

Everything you should know for AP Calculus...

1. Limit Definition of the Derivative: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
2. Limit Definition of the Derivative (Alternative Form): $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$
3. Average Rate of change of $f(x)$ on $[a, b]$: $\frac{f(b) - f(a)}{b - a}$
4. Average Value of $f(x)$ on $[a, b]$: $\frac{1}{b - a} \int_a^b f(x) dx$
5. Intermediate Value Theorem:
 - Conditions: $f(x)$ is continuous on the closed interval, $[a, b]$
 - Conclusion: There is a value c such that $f(a) \leq f(c) \leq f(b)$ or $f(b) \leq f(c) \leq f(a)$ and $a \leq c \leq b$.
6. Rolle's Theorem:
 - Conditions: $f(x)$ is continuous on the closed interval, $[a, b]$, and differentiable on the open interval (a, b) and $f(a) = f(b)$
 - Conclusion: $f'(c) = 0$ and $a < c < b$
7. Mean Value Theorem:
 - Conditions: $f(x)$ is continuous on the closed interval, $[a, b]$, and differentiable on the open interval (a, b)
 - Conclusion: $f'(c) = \frac{f(b) - f(a)}{b - a}$ and $a < c < b$
8. Extreme Value Theorem:
 - Conditions: $f(x)$ is continuous on the closed interval, $[a, b]$
 - Conclusion: $f(x)$ has an absolute maximum and absolute minimum at a critical number or an endpoint on $[a, b]$
9. Double Angle Identities:
 - $\sin 2x = 2 \sin x \cos x$
 - $\cos 2x = \cos^2 x - \sin^2 x$
10. Power Reducing Identities:
 - $\sin^2 x = \frac{1 - \cos 2x}{2}$
 - $\cos^2 x = \frac{1 + \cos 2x}{2}$
11. Critical Number: f has a critical number when $f' = 0$ or is undefined
12. Increasing/Decreasing:
 - f is increasing when $f' > 0$
 - f is decreasing when $f' < 0$
13. Concavity:
 - f is concave up when f' is increasing and $f'' > 0$
 - f is concave down when f' is decreasing and $f'' < 0$
14. Relative Extrema (1st Derivative Test):
 - f has a relative maximum when f' changes from positive to negative.
 - f has a relative minimum when f' changes from negative to positive.

28. Tangent Line Approximations

1. Write the tangent line at the given point: $(a, f(a))$

$$y - f(a) = f'(a)(x - a)$$

2. Then plug in the point $x = x_1$

$$y = f'(a)(x_1 - a) + f(a)$$

29. Absolute extrema – Compare the y-values of the relative extrema AND the endpoints. If there is only 1 critical number then the critical number is both a relative and absolute extrema.

30. Particle Motion - Position/ Velocity/ Acceleration

- PVAJ:

- Position: $x(t)$

- Velocity: $x'(t) = v(t)$

- Acceleration: $x''(t) = v'(t) = a(t)$

- SPEED

- Speed: $|v(t)|$

- INCREASING – velocity and acceleration have the same signs

- DECREASING – velocity and acceleration have opposite signs

- Initially: $t=0$

- At Rest: $v(t)=0$

- Particle Moving Right: $v(t)>0$

- Particle Moving Left: $v(t)<0$

- Total Distance on $[a, b]$: $\int_a^b |v(t)| dt$

- Average velocity on $[a, b]$: $\frac{x(b) - x(a)}{b - a}$ or $\frac{1}{b - a} \int_a^b v(t) dt$

- Instantaneous velocity at $t=a$: $v(a) = x'(a)$

31. Derivative Formulas

$$\frac{d}{dx}[c] = 0$$

$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[cx] = c$$

$$\frac{d}{dx}[x^c] = cx^{c-1}$$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$

32. Integration Formulas

$$\int dx = x + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{dx}{x} = \ln|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = -\ln|\cos x| + c$$

$$\int \csc x dx = -\ln|\csc x + \cot x| + c$$

$$\int \sec x dx = \ln|\sec x + \tan x| + c$$

$$\int \cot x dx = \ln|\sin x| + c$$

$$\int \sec^2 x dx = \tan x$$

$$\int \csc^2 x dx = -\cot x + c$$

$$\int \sec x \tan x dx = \sec x + c$$

$$\int \csc x \cot x dx = -\csc x + c$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + c$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + c$$

1. What is $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$?

- (A) 1
- (B) $\frac{\sqrt{2}}{2}$
- (C) 0
- (D) -1
- (E) The limit does not exist.

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$f(x) = \cos x$$

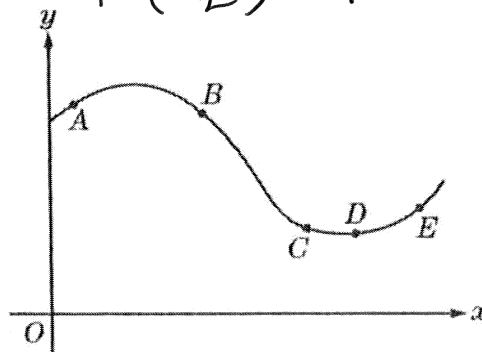
$$f'(x) = -\sin x$$

$$f'\left(\frac{3\pi}{2}\right) = 1$$

2. At which of the five points on the graph in the figure

at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

- (A) A
- (B) B
- (C) C
- (D) D
- (E) E



3. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

- (A) $-\frac{3}{2}$
- (B) -1
- (C) $-\frac{5}{14}$
- (D) $-\frac{3}{14}$
- (E) 0