Rates of change and Rectilinear Motion

- Application of the Derivative: slope, rate of change, velocity

- Rate of Change(definition): change in position ÷ change in time

Examples:

- m/sec
- rad/sec
- m^2/sec
-
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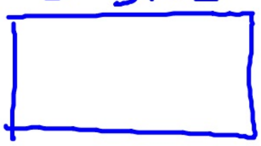
Ex 1: The length of a rectangle is given by $3t+2$ meters. The width is 3 times the length. Find the rate of change of the area with respect to time at t = 1hr.

$L = 3t+2$



$W = 3L$

$3(3t+2)$

$$A = LW$$
$$A = (3t+2)(3(3t+2))$$
$$A = 3(3t+2)^2$$
$$A' = 6(3t+2)^1 \cdot 3$$
$$A'(1) = 6(5)(3)$$
$$= 90\ m^2/hr$$

Ex2: unit in answer bacteria/hr

Ex 2: A population of 500 bacteria is introduced into a culture and grows in numbers according to the equation $P(t) = 500\left(1 + \frac{4t}{50 + t^2}\right)$, where t is measured in hours. Find the rate at which the population is growing when $t = 2$.

- Rectilinear Motion Problems – When we talk about these types of problems, we often talk about three types of functions.

1. Position: meters

   Notation: $s(t)$ or $x(t)$

   Application(s): where is the object after 3 sec.

2. Velocity: meters/sec

   1st Deriv.

   Notation: $s'(t)$, $x'(t)$, or $v(t)$

   Application(s): find the velocity at $t = 3$ sec.

3. Acceleration: meters/sec^2

   2nd deriv.

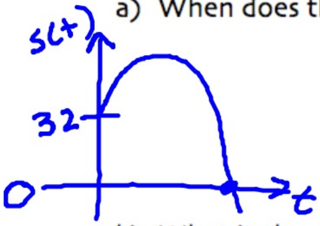
   Notation: $s''(t)$, $x''(t)$, $v'(t)$, $a(t)$

   Application(s): find acceleration at $t = 3$ sec

PVAJ

Ex 3: At time t=0 seconds a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$.

a) When does the diver hit the water?



$0 = -16t^2 + 16t + 32$

$0 = -16(t^2 - t - 2)$

$0 = -16(t-2)(t+1)$

$t = 2$ sec

b) What is the diver's velocity at impact?

↑ instantaneous velocity

$s'(t) = -32t + 16$

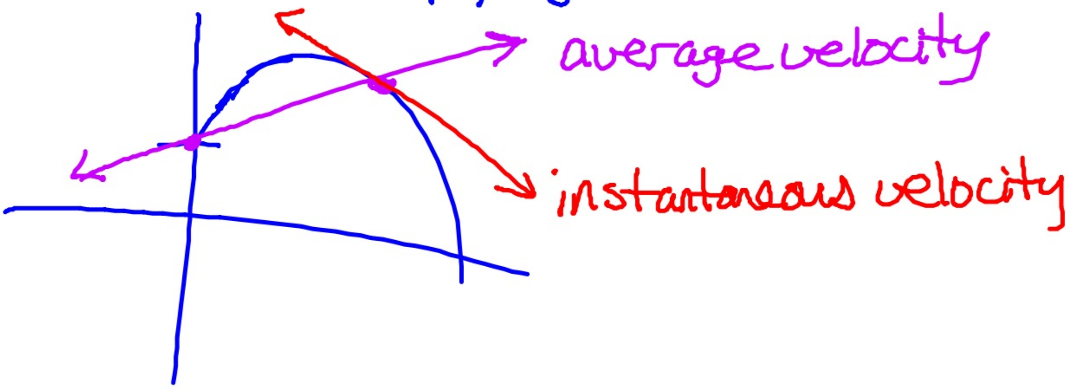
$s'(2) = -32(2) + 16$

$= -48$ ft/sec

* Average Velocity vs. Instantaneous Velocity
  - Average Velocity: change in position ÷ change in time
    $\Delta y / \Delta x$ (slope of a line ; slope of secant)

    Formula: $\frac{y_2 - y_1}{x_2 - x_1}$ $\frac{s(b) - s(a)}{b - a}$
  - Instantaneous Velocity: slope of tangent line,
    velocity at a certain time t

    Formula: $s'(t)$; plug in t



- Speed: distance/time $|v(t)|$

- Rest: velocity = 0 $v(t) = 0$

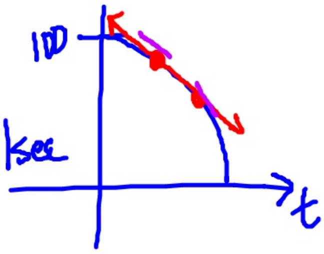
- Left and Right Motion:
  - Left: $v(t)$ negative ; $v(t) < 0$
  - Right: $v(t)$ positive ; $v(t) > 0$
  - Changes Direction: when $v(t)$ changes signs (direction)

Ex 4: A billiard ball is dropped from a height of 100 ft, its height s at time t is given by the position function $s(t) = -16t^2 + 100$, where s is measured in feet and t is measured in seconds.

a) Find the average velocity over time interval [1, 2].

$(1, 84) \quad (2, 36)$

$$\frac{s(2)-s(1)}{2-1} = \frac{36-84}{2-1} = -48 \text{ ft/sec}$$



b) Find the Instantaneous velocity at the endpoints of the interval.

$s'(t) = -32t$

$s'(1) = -32$ ft/sec

$s'(2) = -64$ ft/sec

c) Find the speed at the endpoints of the interval.

$|s'(1)| = 32$ ft/sec

$|s'(2)| = 64$ ft/sec

Ex 5: A particle starts at time t=o and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the velocity of the particle at any time $t \geq 0$. Simplify.

$x'(t) = (t-1)^2(8t-11)$

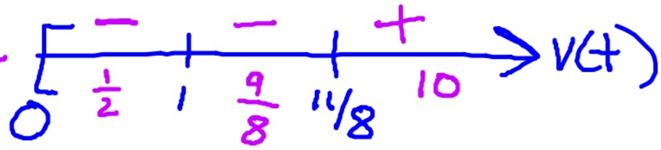
b) Determine the values of t for which the particle is at rest. Justify

when $v(t) = 0$

$0 = (t-1)^2(8t-11)$

$t = 1, 11/8$ because $v(t) = 0$ at these times

c) Determine the values of t for which the particle is moving to the left. JYA.



moving to the left on $(0,1) \cup (1, 11/8)$ because $v(t) < 0$ on these intervals

e.) $t = 11/8$ because $v(t)$ changes signs