Evaluate the limit

Two types of questions:

Definition of derivative (h approaches zero)

Alternate form of derivative (x approaches a value)

Definition of derivative

8 $\lim_{h \to 0} \frac{\tan(2(x+h)) - \tan(2x)}{h} = 2\sec^2(2x)$

$f(x) = \tan(2x)$

$f'(x) = \sec^2(2x) \cdot 2$

$= 2\sec^2(2x)$

11.) $$\lim_{\Delta x \to 0} \frac{\left((x+\Delta x)^3+1\right)-(x^3+1)}{\Delta x} = 3x^2$$

$f(x) = x^3+1$

$f'(x) = 3x^2$

Alternate form of the derivative $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

5 $\lim_{x \to 2} \frac{(x^3 + 6x) - 20}{x - 2} = 18$

$f(x) = x^3 + 6x$

$f'(x) = 3x^2 + 6$

$f'(2) = 18$

7 $\lim_{x \to 0} \frac{\sec x - 1}{x} = 0$

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f'(0) = 0$

10.) $s(t) = \frac{t^4}{4} - \frac{t^3}{6} - t^2 + 1$

$s'(t) = t^3 - \frac{1}{2}t^2 - 2t$

$s''(t) = 3t^2 - t - 2$

$0 = (3t+2)(t-1)$

a.) $t = 1$

b.) $s(1) = \frac{1}{4} - \frac{1}{6} - 1 + 1$

$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

P

V

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12.) $x(t) = 2\cos\left(\frac{\pi}{2}t^2\right)$

$$x'(t) = -2\sin\left(\frac{\pi}{2}t^2\right)\cdot(\pi t)$$

$$\boxed{x'(t) = -2\pi t\sin\left(\frac{\pi}{2}t^2\right)}$$

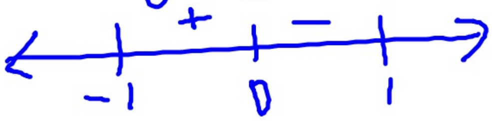
$$x''(t) = -2\pi\left(t\cos\left(\frac{\pi}{2}t^2\right)\pi t + \sin\left(\frac{\pi}{2}t^2\right)\cdot 1\right)$$

$$= -2\pi\left(\pi t^2\cos\left(\frac{\pi}{2}t^2\right) + \sin\left(\frac{\pi}{2}t^2\right)\right)$$

$0 = -2\pi t\sin\left(\frac{\pi}{2}t^2\right)$ $\quad t = 0 \quad \sin\left(\frac{\pi}{2}t^2\right) = 0$

$0 = t\sin\left(\frac{\pi}{2}t^2\right)$

$t = 0$



$y = \cos(2x)$ $\qquad [0, 2\pi]$

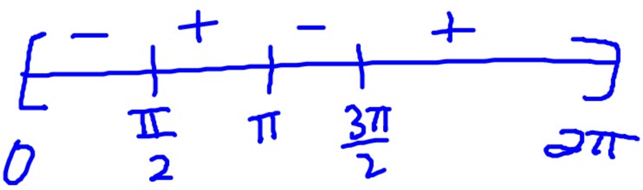
When is the particle moving to the right?

$$\boxed{y' = -2\sin(2x)}$$

$$0 = -2\sin(2x)$$

$$0 = \sin(2x)$$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$



2.) $V = \pi r^2 h$

$V = \pi \left(\sqrt{t+2}\right)^2 \frac{1}{2}\sqrt{t}$

$V = \frac{\pi}{2}(t+2)\sqrt{t}$

$V = \frac{\pi}{2}\left(t^{3/2} + 2t^{1/2}\right)$

$V' = \frac{\pi}{2}\left(\frac{3}{2}t^{1/2} + \frac{2}{2}t^{-1/2}\right)$

$V' = \frac{\pi}{2}\left[\frac{1}{2}t^{-1/2}(3t+2)\right]$

$= \frac{\pi(3t+2)}{4\sqrt{t}}$