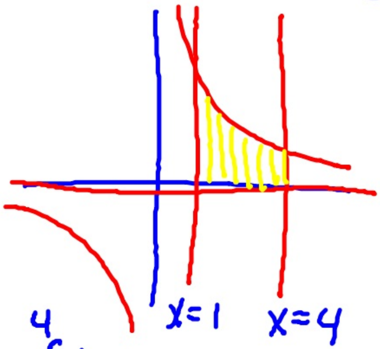
Let R be the region bounded by $y = 1/x$, $x = 1$, $x = 4$, and the x-axis.

The vertical line $x = k$ divides region R into two regions of equal area. Set up, but do not solve, an equation involving one or more integrals whose solution gives the value of k.



$$\int_1^4 \frac{1}{x}dx = \ln 4$$

$$\frac{\ln 4}{2} = \int_1^k \frac{1}{x}dx$$

$$\frac{\ln 4}{2} = \int_k^4 \frac{1}{x}dx$$

$$\int_1^k \frac{1}{x}dx = \int_k^4 \frac{1}{x}dx$$

$$\int_{0}^{5} x\,dx = \int_{0}^{1} x\,dx + \int_{1}^{5} x\,dx$$

$$\int_{0}^{5} x\,dx \neq \int_{0}^{k} x\,dx + \int_{k}^{5} x\,dx$$

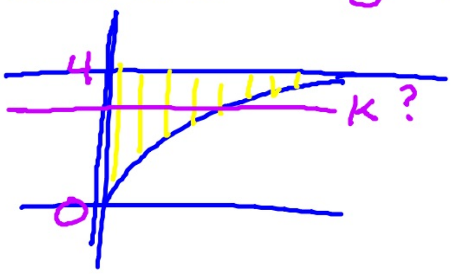
Mock 2
Thursday, April 7th (during class)
Friday, April 8th (during class)
Monday, April 11th (after school)

Mock 3
Saturday, April 23rd

Let R be the region bounded by $y = \sqrt{x}$, y = 4, and the y-axis.

The horizontal line y = k divides region R into two regions of equal area. Set up, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$y = \sqrt{x}$; $x = y^2$



$$\int_0^k y^2\,dy = \int_k^4 y^2\,dy$$

$$\int_0^4 y^2\,dy$$

$$\frac{1}{3}y^3 \Big|_0^4$$

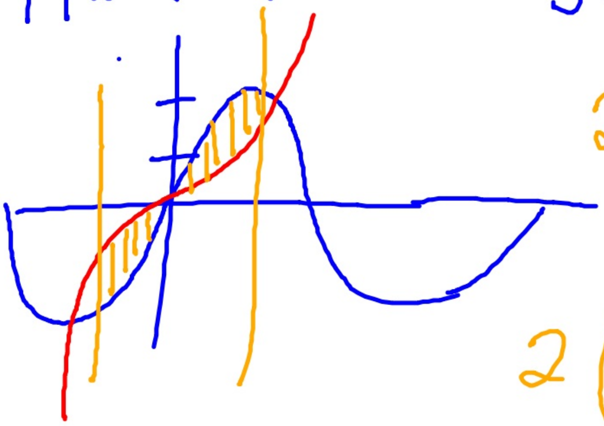
$$\frac{64}{3}$$

$$\int_0^k y^2\,dy = \frac{32}{3}$$

$$\frac{1}{3}k^3 = \frac{32}{3} \ ; \ k^3 = 32$$

$$k = \sqrt[3]{32}$$

49.) $f(x)=2\sin x \quad g(x)=\tan x \quad \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$



$$2\int_0^{\pi/3}(2\sin x-\tan x)\,dx$$

$$2\left(-2\cos x+\ln|\cos x|\right)\Big|_0^{\pi/3}$$

$$2\left(\left(-1+\ln\tfrac{1}{2}\right)-\left(-2+0\right)\right)$$

$$-2+2\ln\tfrac{1}{2}+4 = 2+2\ln\tfrac{1}{2}$$

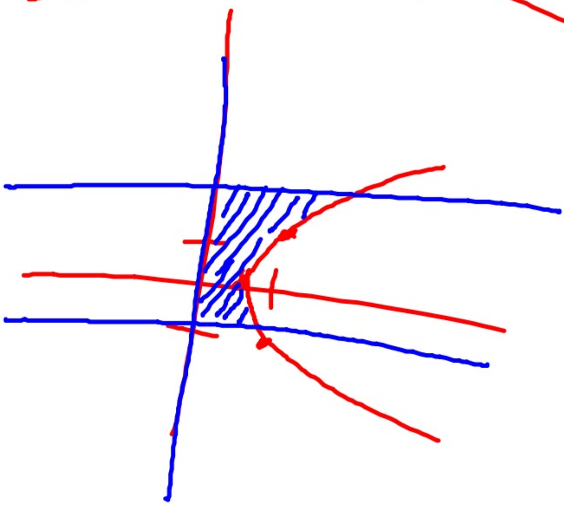
37.) $f(x) = x(x^2 - 3x + 3)$ $g(x) = x^2$

$$\int_0^1 (f(x) - g(x))\,dx + \int_1^3 (g(x) - f(x))\,dx$$

33) $f(y) = y^2+1$ $g(y)=0$ $y=-1$, $y=2$

$x=0$

$x=y^2+1$



x	y
2	-1
1	0
2	1

$$\int_{-1}^{2}(y^2+1-0)\,dy$$

7.2: Volume by cross section

You will be given:

- base (determined by region enclosed by functions)
- geometric shape
- perpendicular to x-axis or y-axis

Geometric Shapes

Square s^2

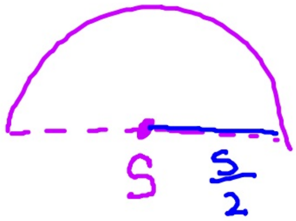
Rectangle $s \cdot l$

Equilateral triangle $\frac{\sqrt{3}}{4} s^2$

Isosceles right triangle (hyp. on base) $\frac{1}{4} s^2$

Isosceles right triangle (leg on base) $\frac{1}{2} s^2$

Semi-circles $\frac{\pi}{8} s^2$

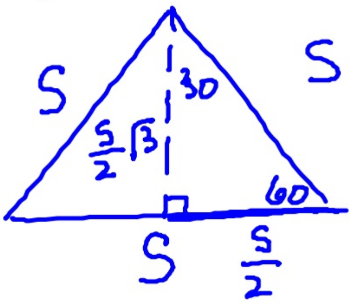


$\frac{1}{2}\pi r^2$

$\frac{1}{2}\pi\left(\frac{s}{2}\right)^2$

$\frac{\pi}{8} s^2$

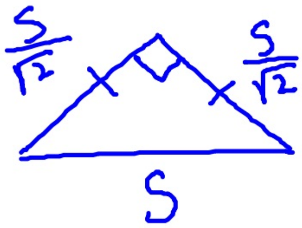
equilateral Δ



$$\frac{1}{2}(S)\left(\frac{S\sqrt{3}}{2}\right)$$

$$\frac{S^2\sqrt{3}}{4}$$

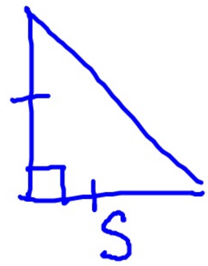
Isosc Δ
hyp. on
base



$$\frac{1}{2}\left(\frac{S}{\sqrt{2}}\right)\left(\frac{S}{\sqrt{2}}\right)$$

$$\frac{1}{4}S^2$$

Isosc. Δ
leg on base



$$\frac{1}{2}(S)(S)$$

$$\frac{1}{2}S^2$$

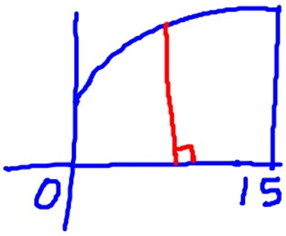
To find the volume of a cross section, integrate the area of the geometric shape on the interval [a, b]

$\perp$ x-axis $\int_a^b A(x)\,dx$; $A(x)$ = Area of geometric shape

$\perp$ y-axis $\int_a^b A(y)\,dy$; $A(y)$ = Area of geometric shape

① $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

perpendicular to the x-axis (squares)



$$\int_0^{15} A(x)\,dx$$

$$\int_0^{15} (2\sqrt{x} + 5)^2\,dx$$

$= 1599.596$

or

1599.597

$S = \text{top} - \text{bottom}$

$S = 2\sqrt{x} + 5 - 0$

$S^2 = (2\sqrt{x} + 5)^2$

$A(x) = (2\sqrt{x} + 5)^2$

2 $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

Equilateral triangles perpendicular to the x-axis

$$\frac{\sqrt{3}}{4}\int_0^{15} (2\sqrt{x}+5)^2\,dx$$

692.645

or

692.646

$S = \text{Top} - \text{Bottom}$

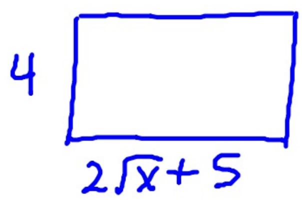
$S^2 = (2\sqrt{x}+5)^2$

$A(x) = \frac{\sqrt{3}}{4} S^2$

$= \frac{\sqrt{3}}{4}(2\sqrt{x}+5)^2$

3 $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

Rectangle with height 4 (perpendicular to x-axis)



$$A(x) = 4\left(2\sqrt{x} + 5\right)$$

$$4\int_{0}^{15} \left(2\sqrt{x} + 5\right) dx$$

609.838 or
609.839

4 $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

Rectangle with height twice the length
(perpendicular to x-axis)

$h = 2l$

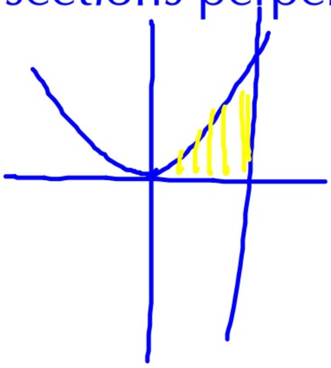
$h = 2(2\sqrt{x} + 5)$

$A(x) = 2(2\sqrt{x} + 5)^2$

$$2\int_0^{15} (2\sqrt{x} + 5)^2 \, dx$$

3199.193

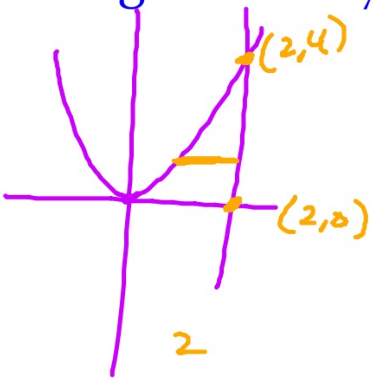
4) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the x-axis are semicircles.



$S = T - B = x^2 - 0$ $\quad \frac{\pi}{8}s^2$

$$\frac{\pi}{8}\int_0^2 x^4\,dx = \frac{4\pi}{5} = 2.513$$

4) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the y-axis are isosceles triangles with hypotenuse on base.



$$\int_0^4 \frac{1}{4}\left(2-\sqrt{y}\right)^2 dy$$

$$\frac{2}{3}$$

S = Right − left

$S = 2 - \sqrt{y}$

$y = x^2$

$\sqrt{\cancel{x}y} = x$