AP Calculus Chapter 5 Free Response Questions (No Calculator)

1)

A particle starts at the point $(5,0)$ at $t=0$ and moves along the x-axis in such a way that at time $t>0$ its velocity $v(t)$ is given by $v(t)=\frac{t}{1+t^2}$.

(a) Determine the maximum velocity attained by the particle. Justify your answer.

(b) Determine the position of the particle at $t=6$.

(c) Find the limiting value of the velocity as t increases without bound.

2)

A particle moves along the x-axis in such a way that its acceleration at time t for $t>0$ is given by $a(t)=\frac{3}{t^2}$. When $t=1$, the position of the particle is 6 and the velocity is 2.

(a) Write an equation for the velocity, $v(t)$, of the particle for all $t>0$.

(b) Write an equation for the position, $x(t)$, of the particle for all $t>0$.

(c) Find the position of the particle when $t=e$.

3)

Let f be the function defined by $f(x)=-2+\ln(x^2)$.

(a) For what real numbers x is f defined?

(b) Find the zeros of f.

(c) Write an equation for the line tangent to the graph of f at $x=1$.

4)

A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$. At time $t = 1$, the position of the particle is $x(1) = 6$.

(a) Write an expression for the acceleration of the particle.

(b) For what values of t is the particle moving to the right?

(c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

5)

Given the function f defined by $f(x) = \ln(x^2 - 9)$.

(a) Describe the symmetry of the graph of f.

(b) Find the domain of f.

(c) Find all values of x such that $f(x) = 0$.

(d) Write a formula for $f^{-1}(x)$, the inverse function of f, for $x > 3$.

6)

Let f and g and their inverses f^{-1} and g^{-1} be differentiable functions and let the values of f, g, and the derivatives f' and g' at $x = 1$ and $x = 2$ be given by the table below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	5	4
2	2	π	6	7

Determine the value of each of the following.

(a) The derivative of $f + g$ at $x = 2$

(b) The derivative of fg at $x = 2$

(c) The derivative of $\frac{f}{g}$ at $x = 2$

(d) $h'(1)$ where $h(x) = f(g(x))$

(e) The derivative of g^{-1} at $x = 2$

7)

A particle moves along the x-axis in such a way that at time $t > 0$ its position coordinate is $x = \sin(e^t)$.

(a) Find the velocity and acceleration of the particle at time t.

(b) At what time does the particle first have zero velocity?

(c) What is the acceleration of the particle at the time determined in part (b)?

8)

A function f is defined by $f(x) = xe^{-2x}$ with domain $0 \leq x \leq 10$.

(a) Find all values of x for which the graph of f is increasing and all values of x for which the graph is decreasing.

(b) Give the x- and y-coordinates of all absolute maximum and minimum points on the graph of f. Justify your answers.

9)

A particle moves on the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 2te^{-t}$.

(a) Find the acceleration of the particle at $t = 0$.

(b) Find the velocity of the particle when its acceleration is 0.

10)

The acceleration of a particle moving along a straight line is given by $a = 10e^{2t}$.

(a) Write an expression for the velocity v, in terms of time t, if $v = 5$ when $t = 0$.

(b) During the time that the velocity increases from 5 to 15, how far does the particle travel?

(c) Write an expression for the position s, in terms of time t, of the particle if $s = 0$ when $t = 0$.

11)

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

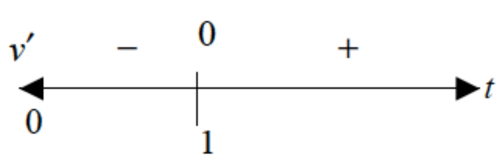
(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

Answers

1 (a) The velocity has a maximum value of ½. (b) $s(6) = 5 + \ln\sqrt{37}$ (c) 0	2 (a) $v(t) = \int a(t)\,dt = \int \frac{3}{t^2}dt = -\frac{3}{t} + C$ $2 = v(1) = -3 + C$ Therefore $C = 5$ and so $v(t) = -\frac{3}{t} + 5$. (b) $x(t) = \int v(t)\,dt = \int (-\frac{3}{t} + 5)dt = -3\ln t + 5t + C$ $6 = x(1) = -3\ln 1 + 5 + C$ Therefore $C = 1$ and so $x(t) = -3\ln t + 5t + 1$. (c) $x(e) = -3\ln e + 5e + 1 = -3 + 5e + 1 = 5e - 2$
3. (a) f(x) is defined everywhere except x = 0 (b) The zeros are $\pm e$ (c) y = 2x - 4	4. (a) $a(t) = v'(t) = \ln t + t \cdot \frac{1}{t} - 1 = \ln t$ (b) $v(t) = t\ln t - t > 0$ $t(\ln t - 1) > 0$ $t > e$ (c) $v'(t) = \ln t = 0$ $t = 1$  minimum velocity is $v(1) = -1$

5.

(a) $f(-x) = \ln((-x)^2 - 9) = \ln(x^2 - 9) = f(x)$
Therefore the graph of f is symmetric with respect to the y-axis.

(b) Since we need $x^2 - 9 > 0$, the domain of f is the set $\{x \mid x < -3 \text{ or } x > 3\}$

(c) $f(x) = 0$ when $x^2 - 9 = 1$. This happens for $x = \pm\sqrt{10}$.

(d) Method 1:

$f(x) = \ln(x^2 - 9) \Rightarrow x^2 - 9 = e^{f(x)} = e^y$
Since $x > 3$, $x = \sqrt{e^y + 9}$.

Hence $f^{-1}(x) = \sqrt{e^x + 9}$.

6.

(a) $(f+g)'(2) = f'(2) + g'(2) = 6 + 7 = 13$

(b) $(fg)'(2) = f(2)g'(2) + f'(2)g(2) = 2 \cdot 7 + 6 \cdot \pi = 14 + 6\pi$

(c) $\left(\frac{f}{g}\right)'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \frac{\pi \cdot 6 - 2 \cdot 7}{\pi^2} = \frac{6\pi - 14}{\pi^2}$

(d) $(f \circ g)'(1) = f'(g(1))g'(1) = f'(2) \cdot 4 = 6 \cdot 4 = 24$

(e) $h = g^{-1}(g(h))$. Therefore $1 = (g^{-1})'(g(h)) \cdot g'(h)$. Let $h = 1$. Then

$1 = (g^{-1})'(2) \cdot g'(1)$

$(g^{-1})'(2) = \frac{1}{g'(1)} = \frac{1}{4}$

7)

(a) $x = \sin(e^t)$

$v = \frac{dx}{dt} = e^t \cos(e^t)$

$a = \frac{dv}{dt} = e^t(\cos(e^t) - e^t \sin(e^t))$

(b) $v(t) = 0$ when $\cos(e^t) = 0$. Hence $e^t = \frac{\pi}{2}$ gives the first time when the velocity is zero, and so $t = \ln\frac{\pi}{2}$.

(c) $a\left(\ln\frac{\pi}{2}\right) = \frac{\pi}{2}\left(\cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}\right) = -\frac{\pi^2}{4}$

8)

a. increasing on [0, ½]
decreasing on [½ , 10]

b. Abs max: $\left(\frac{1}{2}, \frac{1}{2e}\right)$

abs min. (0, 0)

9)

a. -4

b. $\frac{-2}{e^2}$

10)

a. $v = 5e^{2t}$

b. 5

c. $s = \frac{5}{2}e^{2t} - \frac{5}{2}$

11)

(a) $a(t) = v'(t) = -e^{1-t}$

$a(3) = -e^{-2}$

(b) $a(3) < 0$

$v(3) = -1 + e^{-2} < 0$

Speed is increasing since $v(3) < 0$ and $a(3) < 0$.

(c) $v(t) = 0$ when $1 = e^{1-t}$, so $t = 1$.

$v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$.

Therefore, the particle changes direction at $t = 1$.

(d)

$x(t) = -t - e^{1-t}$

$x(0) = -e$

$x(1) = -2$

$x(3) = -e^{-2} - 3$

$$\begin{aligned} \text{Distance} &= (x(1) - x(0)) + (x(1) - x(3)) \\ &= (-2 + e) + \left(1 + e^{-2}\right) \\ &= e + e^{-2} - 1 \end{aligned}$$