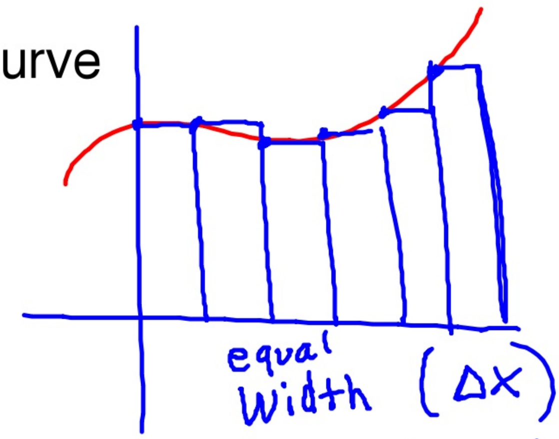
Left, Right, Midpoint and Trapezoidal Sums

Approximating area under a curve

$$\int_a^b f(x)\,dx$$



n: number of rectangles

$n \to \infty$; $\Delta x \to 0$

If there is a constant width, the width can be calculated by:

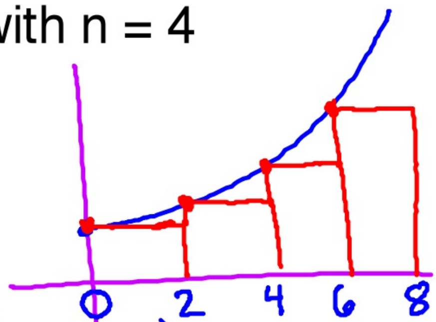
Width = (b - a)/n

$[\underset{a}{0}, \underset{b}{12}]$ $n = 4$

$w = \frac{12-0}{4} = 3$

Left sum: Start at the LEFT of the interval $w = \frac{8-0}{4} = 2$

$f(x) = x^2 + 1$ on the interval [0, 8] with n = 4

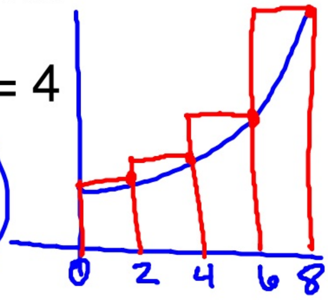


$$\int_0^8 f(x)\,dx \approx 2\cdot f(0) + 2\cdot f(2) + 2\cdot f(4) + 2\cdot f(6)$$

$$\approx 2(1 + 5 + 17 + 37)$$

$$\approx 120$$

Right Sum: start at the right of the interval



f(x) = x^2 + 1 for the interval [0, 8] with n = 4

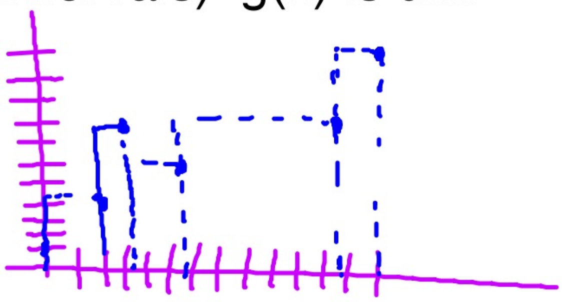
$$\int_0^8 f(x)\,dx \approx 2\left(f(8)+f(6)+f(4)+f(2)\right)$$

$$\approx 2\left(65+37+17+5\right)$$

$$\approx 248$$

Right sum with a chart (5 sub-intervals) g(x) is diff.

X	0	2	3	7	12	13
f(x)	1	4	8	5	8	11



$$\int_0^{13} f(x)\,dx \approx 1f(13) + 5f(12) + 4f(7) + 1f(3) + 2f(2)$$

$$1\cdot 11 + 5\cdot 8 + 4\cdot 5 + 1\cdot 8 + 2\cdot 4$$

$$\approx 87$$

Midpoint Sums: Find the midpoints of the sub-intervals

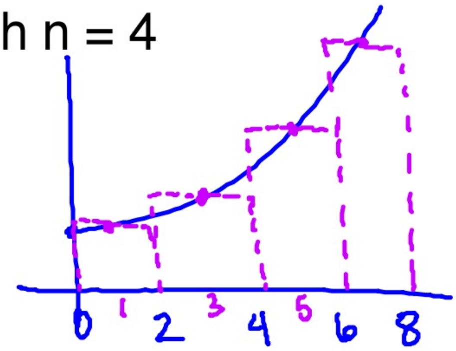
f(x) = x^2 + 1 on the interval [0, 8] with n = 4

$$\int_0^8 f(x)\,dx \approx$$

$$2\left(f(1)+f(3)+f(5)+f(7)\right)$$

$$2\left(2+10+26+50\right)$$

$$176$$



Midpoint with a chart (3 sub-intervals)

x	0	4	8	12	16	20	24
$g(x)$	1	5	7	8	10	9	13

$$\Delta x = \frac{b-a}{n}$$

$$= \frac{24-0}{3}$$

$$\Delta x = 8$$

$$\int_0^{24} g(x)\,dx \approx 8\left(g(4) + g(12) + g(20)\right)$$

$$8(5 + 8 + 9)$$

$$176$$

Trapezoids: Approximating the area using trapezoids

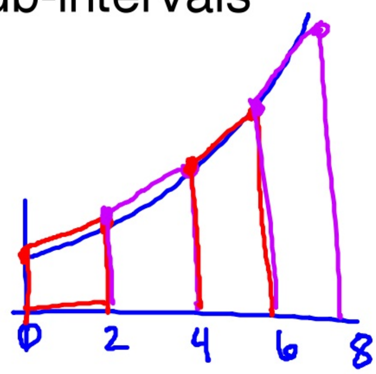
f(x) = x^2 +1 on the interval [0, 8] with 4 sub-intervals

$$\int_0^8 f(x)\,dx \approx$$

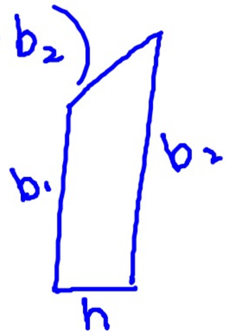
$$\frac{1}{2}(2)\left[f(0)+f(2)+f(2)+f(4)+f(4)+f(6)+f(6)+f(8)\right]$$

$$1(1+5+5+17+17+37+37+65)$$

$$184$$



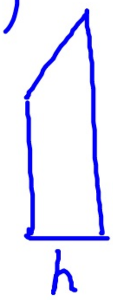
$\frac{1}{2}h(b_1+b_2)$



Trapezoids with a chart (3 sub-intervals)

X	1	3	7	9
g(x)	5	8	4	3

$$\frac{1}{2}h(b_1+b_2)$$

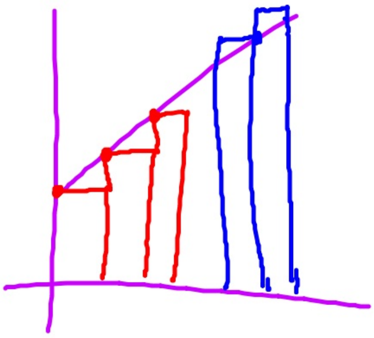


$$\int_1^9 g(x)\,dx \approx \frac{1}{2}\left(2(5+8)+4(8+4)+2(4+3)\right)$$

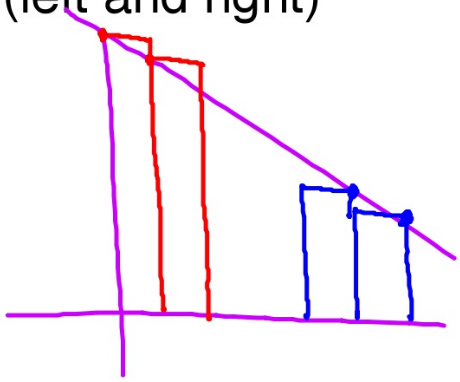
44

Will the estimate be an under or over approximation?

Increasing/Decreasing Functions (left and right)

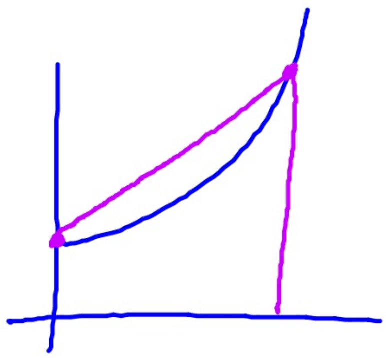


Increasing function
Left Riemann: under
Right Riemann: over

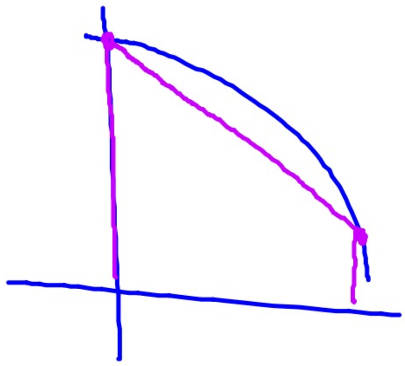


Decreasing function
Left Riemann: over
Right Riemann: under

For Trapezoids: Will the estimate be an under or over approximation for Concave Up/Concave Down functions

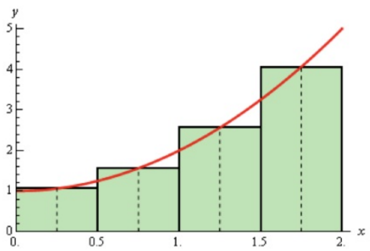


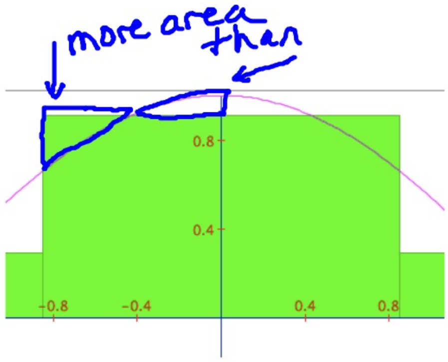
Concave up
Over approx.



Concave down
Under approx.

For Midpoints: Will the estimate be an under or over approximation for Concave Up/Concave Down functions

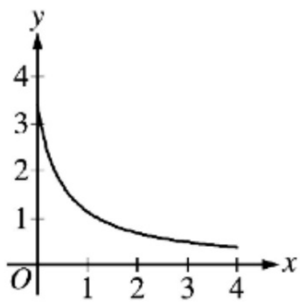


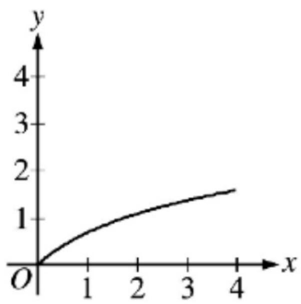


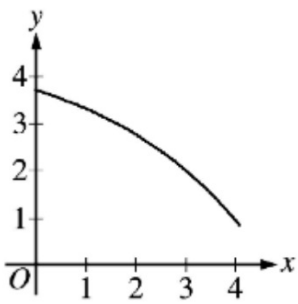
Concave up
Under approx.
more area under
vs. over

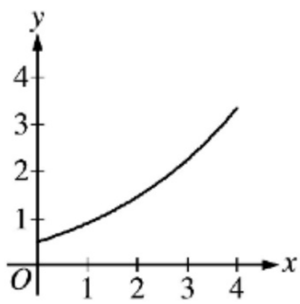
Concave down
Over approx.
More area over
vs. under

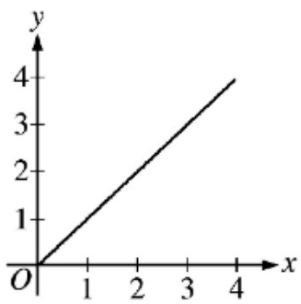
If a trapezoidal sum over approximates $\int_0^4 f(x)dx$, and a right Riemann sum under approximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?











Comments:

If the graph is decreasing, then Right(n) $< \int_a^b f(x)dx <$ Left(n) for the right Riemann and left Riemann sums using n subintervals.

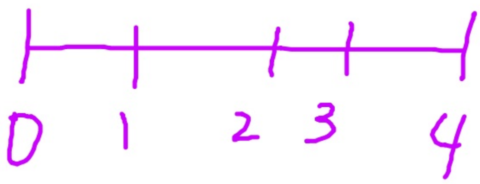
If the graph is concave up, then Mid(n) $< \int_a^b f(x)dx <$ Trap(n) for the trapezoid sum and the midpoint Riemann sum using n subintervals.

Graph (A) is decreasing and concave up, and therefore could be the graph of $y = f(x)$.

If the graph is increasing or concave down, the respective inequalities are reversed.

$f(x)=2x^2+1 \qquad [0,4] \qquad n=4$

$$\int_0^4 (2x^2+1)\,dx \approx$$



$$1\left[f\left(\tfrac{1}{2}\right)+f\left(\tfrac{3}{2}\right)+f\left(\tfrac{5}{2}\right)+f\left(\tfrac{7}{2}\right)\right]$$