AP Calculus
Chapter 3 Free Response Review

1) Omit (b)

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

2)

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

3)

Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

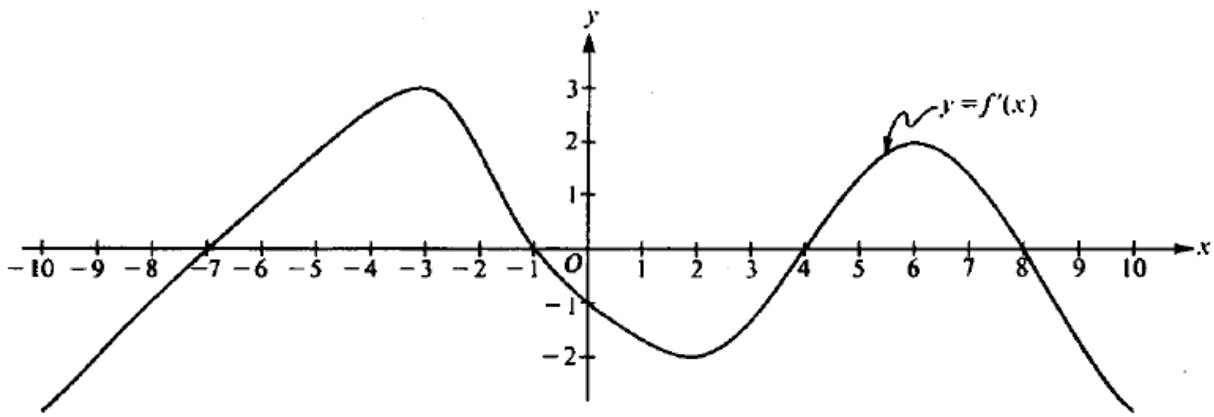
(a) On what intervals is f increasing?

(b) On what intervals is the graph of f concave downward?

(c) Find the value of k for which f has 11 as its relative minimum.

4)

1989 AB5



<u>Note:</u> **This is the graph of the <u>derivative</u> of f, <u>not</u> the graph of f.**

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

(a) For what values of x does the graph of f have a horizontal tangent?

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer.

(c) For value of x is the graph of f concave downward?

Answers

1)

(a) $a(5) \approx \dfrac{v(10) - v(0)}{10 - 0} = \dfrac{0.3}{10} = 0.03 \text{ meters}/\text{sec}^2$

(c) Because $\dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2,$ the Mean Value Theorem implies there is a time t, $40 < t < 60$, such that $v(t) = 2$.

2)

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$$\begin{array}{c|ccccccc} h'(x) & - & 0 & + & \text{und} & - & 0 & + \\ \hline x & & -\sqrt{2} & & 0 & & \sqrt{2} & \end{array}$$

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

(c) $h'(4) = \dfrac{16 - 2}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x - 4)$

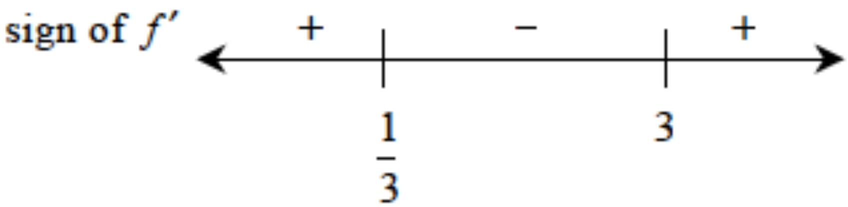
(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

3)

(a) $f'(x) = 3x^2 - 10x + 3$

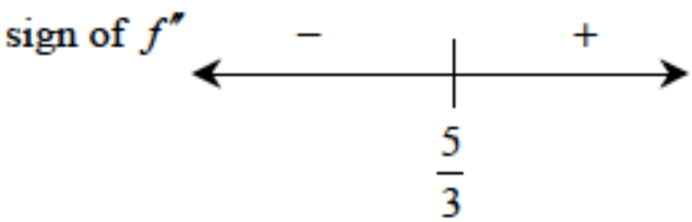
$$= (3x-1)(x-3)$$

$f'(x) = 0$ at $x = \dfrac{1}{3}$ and $x = 3$



f is increasing on $\left(-\infty, \dfrac{1}{3}\right]$ and on $[3, \infty)$

(b) $f''(x) = 6x - 10$



The graph is concave down on $\left(-\infty, \dfrac{5}{3}\right)$

(c) From (a), f has its relative minimum at $x = 3$, so

$$f(3) = (3)^3 - 5(3)^2 + 3(3) + k$$
$$= -9 + k = 11$$
$$k = 20$$

4)

(a) horizontal tangent $\Leftrightarrow f'(x)=0$

$x=-7,-1,4,8$

(b) Relative maxima at $x=-1,\ 8$ because f' changes from positive to negative at these points

(c) f concave downward $\Leftrightarrow f'$ decreasing

$(-3,2),\ (6,10)$