AP Calculus
Chapter 2 FRQ Review

1970 AB1/BC1

Given the parabola $y = x^2 - 2x + 3$:

(a) Find an equation for the line L, which contains the point $(2, 3)$ and is perpendicular to the line tangent to the parabola at $(2, 3)$.

1972 AB2/BC1

A particle starts at time $t = 0$ and moves on a number line so that its position at time t is given by $x(t) = (t-2)^3(t-6)$.

(a) When is the particle moving to the right?

(b) When is the particle at rest?

(c) When does the particle change direction?

1976 AB1

Let f be the real-valued function defined by $f(x) = \sqrt{1+6x}$.

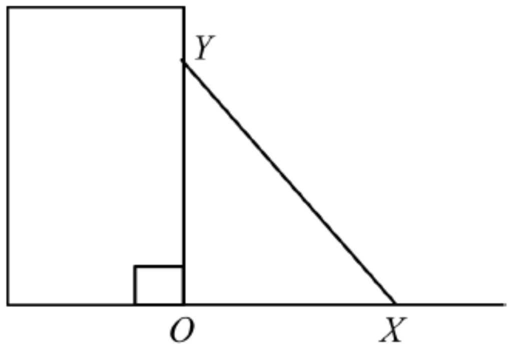
(a) Give the domain and range of f.

(b) Determine the slope of the line tangent to the graph of f at $x = 4$.

(c) Determine the y-intercept of the line tangent to the graph of f at $x = 4$.

(d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$.

1982 AB4

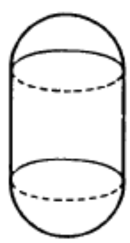


A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

(a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

(b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

1985 AB5/BC2



The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters., the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder is $\pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$).

(a) At this instant, what is the height of the cylinder?

(b) At this instant, how fast is the height of the cylinder increasing?

1992 BC4

Let f be a function defined by $f(x) = \begin{cases} 2x - x^2 & \text{for } x \le 1, \\ x^2 + kx + p & \text{for } x > 1. \end{cases}$

(a) For what values of k and p will f be continuous and differentiable at $x = 1$?

AP Calculus AB–5 / BC–5 **2000**

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

1972 AB 5

Let $y = 2e^{\cos x}$.

(a) Calculate $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

(b) If x and y both vary with time in such a way that y increases at a steady rate of 5 units per second, at what rate is x changing when $x = \dfrac{\pi}{2}$?

Ch 2 FR Review Answers

1970 AB1/BC1
Solution

(a) $y' = 2x - 2$

The slope of the line tangent to the parabola is $m = 2$. Therefore the slope of the line L that is perpendicular to the tangent line is $-\frac{1}{2}$. The equation of the line L is $y - 3 = -\frac{1}{2}(x-2)$, or $y = -\frac{1}{2}x + 4$, or $x + 2y = 8$.

1972 AB2/BC1
Solution

$$x(t) = (t-2)^3(t-6)$$

$$v(t) = x'(t) = (t-2)^3 + 3(t-2)^2(t-6) = (t-2)^2((t-2)+3(t-6))$$
$$= (t-2)^2(4t-20) = 4(t-2)^2(t-5)$$

$$a(t) = x''(t) = 12(t-2)(t-4)$$

(a) The particle is moving to the right when $v(t) = 4(t-2)^2(t-5) > 0$. This happens for $t > 5$.

(b) The particle is at rest when $v(t) = 4(t-2)^2(t-5) = 0$. This happens at $t = 2$ and $t = 5$.

(c) The particle changes direction when the velocity changes sign. The velocity is negative just to the left and just to the right of $t = 2$. The velocity is negative for $t < 5$ and positive for $t > 5$. Therefore the particle only changes direction at $t = 5$.

1976 AB1
Solution

(a) The domain of f is $x \geq -\frac{1}{6}$.

The range of f is $y \geq 0$.

(b) $f'(x) = \frac{3}{\sqrt{1+6x}}$

The slope of the tangent line at $x = 4$ is $f'(4) = \frac{3}{5}$.

(c) $f(4) = 5$

The tangent line is $y - 5 = \frac{3}{5}(x - 4)$

Therefore the y-intercept is at $y = \frac{13}{5}$.

(d) The tangent line parallel to $y = x + 12$ has slope 1.

$$f'(x) = \frac{3}{\sqrt{1+6x}} = 1$$

$$9 = 1 + 6x$$

$$x = \frac{4}{3}$$

$$y = \sqrt{1 + 6\left(\frac{4}{3}\right)} = 3$$

The coordinates of the point are $\left(\frac{4}{3}, 3\right)$.

1982 AB4
Solution

(a) $x^2 + y^2 = 15^2$

Implicit: $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

$$9 \cdot \frac{1}{2} + 12\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{3}{8}$$

(b) $A = \dfrac{1}{2}xy$

Implicit: $\dfrac{dA}{dt} = \dfrac{1}{2}\left(x\dfrac{dy}{dt} + y\dfrac{dx}{dt}\right)$

$$\frac{dA}{dt} = \frac{1}{2}\left(9 \cdot \left(-\frac{3}{8}\right) + 12 \cdot \frac{1}{2}\right)$$

$$\frac{dA}{dt} = \frac{21}{16}$$

1985 AB5/BC2
Solution

(a) $V = \pi r^2 h + \frac{4}{3}\pi r^3$

$144\pi = \pi(3)^2 h + \frac{4}{3}\pi(3)^3$

$h = 12$

At this instant, the height is 12 centimeters.

(b) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r h \frac{dr}{dt} + 4\pi r^2 \frac{dr}{dt}$

$261\pi = \pi(3)^2 \frac{dh}{dt} + 2\pi(3)(12)(2) + 4\pi(3)^2(2)$

$\frac{dh}{dt} = 5$

At this instant, the height is increasing at the rate of 5 centimeters per minute.

1992 BC4

k = -2; p = 2

2000 AB5

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) When $x = 1$, $y^2 - y = 6$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3,\ y = -2$$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

1972 AB 5

(a) $y = 2e^{\cos x}$

$$\frac{dy}{dx} = 2e^{\cos x}(-\sin x) = -2(\sin x)e^{\cos x}$$

$$\frac{d^2y}{dx^2} = -2(\sin x)e^{\cos x}(-\sin x) - 2(\cos x)e^{\cos x} = 2e^{\cos x}(\sin^2 x - \cos x)$$

(b) $\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt} = -2(\sin x)e^{\cos x} \cdot \dfrac{dx}{dt}$

Substituting $\dfrac{dy}{dt} = 5$ and $x = \dfrac{\pi}{2}$ gives

$$5 = -2\left(\sin\frac{\pi}{2}\right)e^{\cos(\pi/2)}\frac{dx}{dt} = -2(1)e^0\frac{dx}{dt} = -2\frac{dx}{dt}.$$

Therefore $\dfrac{dx}{dt} = -\dfrac{5}{2}$ when $x = \dfrac{\pi}{2}$.