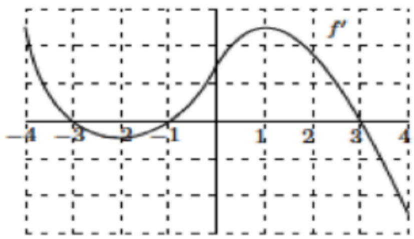
AP Multiple Choice Practice
Chapter 3
Omit Questions with an "X"

1)



The graph of the derivative of a function f is shown above. Which of the following are true about the original function f ?

I. f is increasing on the interval $(-2, 1)$.
II. f is continuous at $x = 0$.
III. f has an inflection point at $x = -2$.

A) I only
B) II only
C) III only
D) II and III only
E) I, II, and III

2)

For what value of k will $x + \dfrac{k}{x}$ have a relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

3)

The *derivative* of $f(x) = \dfrac{x^4}{3} - \dfrac{x^5}{5}$ attains its maximum value at $x =$

(A) -1 (B) 0 (C) 1 (D) $\dfrac{4}{3}$ (E) $\dfrac{5}{3}$

4)

If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

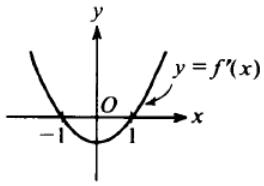
(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$
(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

5)

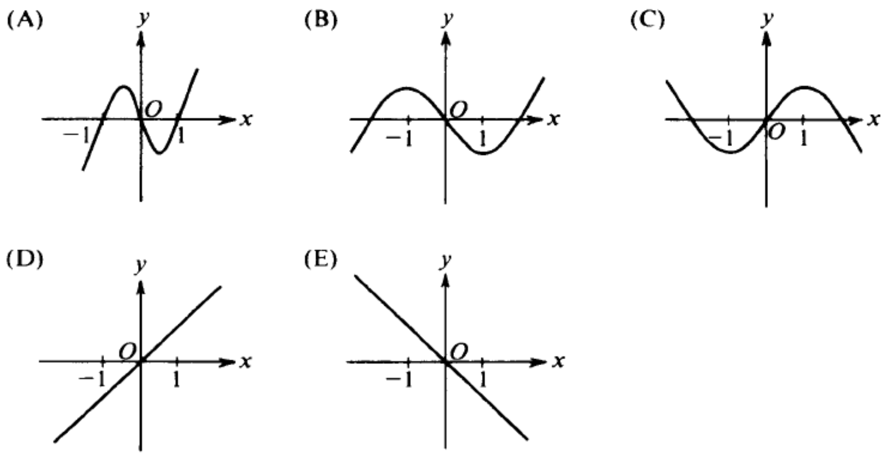
An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$
(D) $y = 3x - 1$ (E) $y = 4x + 1$

6)

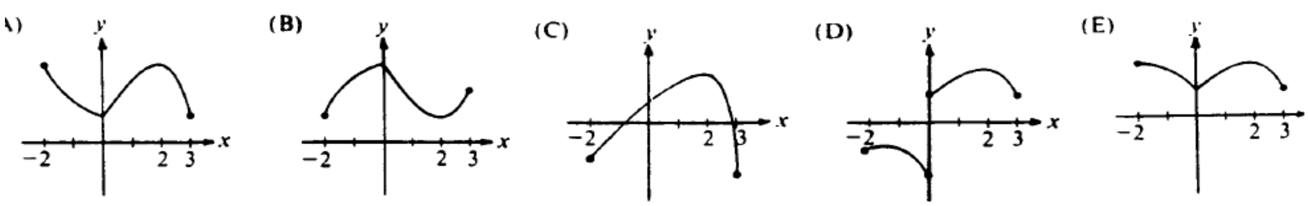


The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

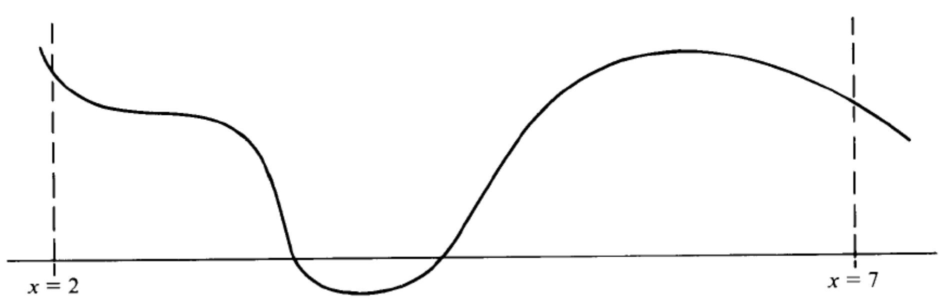


7)

Let f be a function that is continuous on the closed interval $[-2, 3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?



8)



The graph of $y = f(x)$ on the closed interval $[2, 7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

9)

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The minimum value of f on $[3, 7]$ is 12.

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.

III. $f'(x) > 0$ for $5 < x < 7$.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

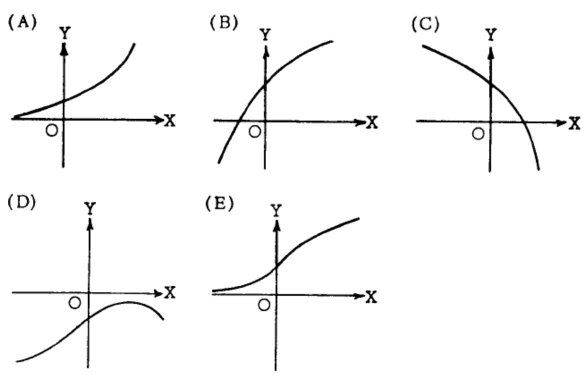
10)
The total number of all relative extrema of the function F whose derivative is $F'(x) = x(x-3)^2(x-1)^4$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

11)
Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$
(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
(C) $-2 < x < 0$ or $x > 2$
(D) $x > \sqrt{2}$
(E) $-2 < x < 2$

12)
If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



X13)

The approximate value of $y=\sqrt{4+\sin x}$ at $x=0.12$, obtained from the tangent to the graph at $x=0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

14)

If $h''(x)=e^{x-1}(2x-1)^2(x-3)^3(4x+5)$ then h(x) has how many points of inflection?

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

X15)

For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

(A) $4+\frac{h}{32}$ (B) $2+\frac{h}{32}$ (C) $\frac{h}{32}$

(D) $4-\frac{h}{32}$ (E) $2-\frac{h}{32}$

X16)

If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

17)

If the graph of $y=x^3+ax^2+bx-4$ has a point of inflection at $(1,-6)$, what is the value of b?

(A) -3 (B) 0 (C) 1 (D) 3

(E) It cannot be determined from the information given.

X18)

The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

19)
The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

20)
The graph of $y=\frac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x<0$ (B) $x<2$ (C) $x<5$ (D) $x>0$ (E) $x>2$

X21)
The first quadrant point on the curve $y^2x=18$ that is closest to the point (2, 0) is

(A) (2, 3) (B) $\left(6,\sqrt{3}\right)$ (C) $\left(3,\sqrt{6}\right)$ (D) $\left(1,3\sqrt{2}\right)$ (E) (3, 2)

22)
At $x=0$, which of the following is true of the function f defined by $f(x)=x^2+e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

23)
If $f(x)=x^2e^x$, then the graph of f is decreasing for all x such that

(A) $x<-2$ (B) $-2<x<0$ (C) $x>-2$ (D) $x<0$ (E) $x>0$

24)
A local minimum value of the function $y=\frac{e^x}{x}$ is

(A) 1/e (B) 1 (C) -1 (D) e (E) 0

25)

For all x in the closed interval [2, 5], the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

26)

The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$.

(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$.

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

X27)
The volume of an open rectangular box is 8 cm^3, and the length of the rectangular base is twice as long as its width. What is the width of the base so that the surface area of the open box is minimized?

(A) $\sqrt[3]{3}$ (B) $\sqrt{3}$ (C) $\sqrt[3]{6}$ (D) 2 (E) $\sqrt{6}$

28)

Which of the following statements is true for $f(x) = \dfrac{1+e^x}{e^x - 1}$?

(A) f(x) has a relative maximum at x = 1
(B) f(x) has a y-intercept at x = 0
(C) f(x) has a root of 0
(D) f(x) is decreasing for all x, $x \neq 0$
(E) f(x) has a vertical asymptote at x = 1

29)
If $f(x) = \ln x$ on the interval [1, e], then what is the value of c on the interval (1, e) that satisfies the Mean Value Theorem?

(A) $\dfrac{1}{e-1}$ (B) $\dfrac{e}{e-1}$ (C) $\dfrac{1+e}{2}$ (D) $e-1$ (E) $e^{\frac{1}{e-1}}$

30)
The maximum acceleration attained on the interval [0,3] by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

Free response Practice

1) A particle moves along a line according to a law of motion $t \geq 0$, where t is measured in seconds and x(t) is measured in meters with $x(t) = t^4 - 18t^2$

a. When does the particle change direction?
b. When is the particle moving to the left?
c. Is the particle slowing down or speeding up at t = 1? Justify.
d. Is the velocity increasing or decreasing at t = 1? Justify.

2) Omit part b

1991 AB 3

Let f be the function defined by $f(x) = (1 + \tan x)^{1.5}$ for $\frac{\pi}{4} < x < \frac{\pi}{2}$.

a. Write an equation for the line tangent to the graph of f at the point where x = 0.
b. Using the equation found in part a, approximate f(0.02).
c. Let $f^{-1}(x)$ denote the inverse function of f. Write an expression that gives $f^{-1}(x)$ for all x in the domain of f^{-1}.

3)

1994 AB 4

A particle moves along the x-axis so that at any time t > 0 its velocity is given by v(t) = t ln t - t.

a. Write an expression for the acceleration of the particle.
b. For what values of t is the particle moving to the right?
c. What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

4)

Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

(a) Find the absolute maximum and minimum values of $f(x)$.

(b) Find the intervals on which f is increasing.

(c) Find the x-coordinate of each point of inflection of the graph of f.

5)

2001 AB 4

Let h be a function defined for all $x \neq 0$ such that h(4) = -3 and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

a. Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
b. On what intervals, if any, is the graph of h concave up? Justify your answer.
c. Write an equation for the line tangent to the graph of h at x = 4.
d. Does the line tangent to the graph of h at x = 4 lie above or below the graph of h for x > 4? Why?

6)

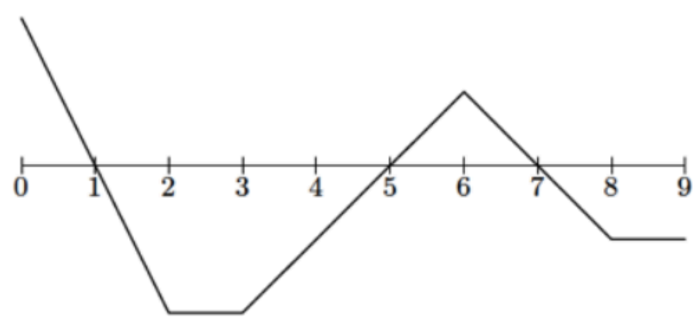
The graph below shows the velocity $v = f(t)$ of a particle moving on a coordinate line.

a) When does the particle move forward? move backward? speed up? slow down?

b) When is the particle's acceleration positive? negative? zero?

c) When does the particle move at its greatest speed?

d) When does the particle stand still for more than an instant?



MC Answers

1	D
2	D
3	C
4	A
5	B
6	B
7	E
8	C
9	B
10	B
11	B
12	B
13	B
14	C
15	B
16	C
17	B
18	D
19	A
20	E
21	C
22	B
23	B
24	D
25	B
26	B
27	C
28	D
29	D
30	D

Free Response Answers

1)

a) t = 3

b) On the interval (0, 3)

c) Speeding up because v(1) < 0 and a(1) < 0 (same sign)

d) Velocity is decreasing because a(1) < 0

2)

(a) $f(0)=1$

$f'(x)=\frac{3}{2}(1+\tan(x))^{1/2}(\sec^2(x))$

$f'(0)=\frac{3}{2}$

$y-1=\frac{3}{2}x$ or $y=\frac{3}{2}x+1$

(b) $f(0.02)\approx\frac{3}{2}(0.02)+1$

(c) $x=(1+\tan y)^{3/2}$

$x^{2/3}=1+\tan y$

$\tan y = x^{2/3}-1$

$y=\arctan(x^{2/3}-1)$

or

$f^{-1}(x)=\arctan(x^{2/3}-1)$

3)

(a) $a(t)=v'(t)=\ln t + t\cdot\frac{1}{t}-1=\ln t$

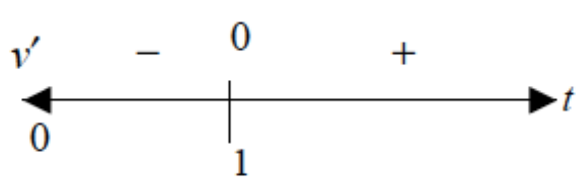
(b) $v(t)=t\ln t - t>0$

$t(\ln t-1)>0$

$t>e$

(c) $v'(t)=\ln t=0$

$t=1$



minimum velocity is $v(1)=-1$

4)

(a) $f'(x)=-e^x\sin x+e^x\cos x$

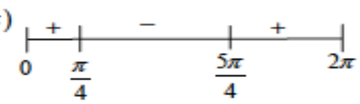
$=e^x[\cos x-\sin x]$

$f'(x)=0$ when $\sin x=\cos x$, $x=\frac{\pi}{4},\frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}e^{5\pi/4}$
2π	$e^{2\pi}$

Max: $e^{2\pi}$; Min: $-\frac{\sqrt{2}}{2}e^{5\pi/4}$

(b) $f'(x)$



Increasing on $\left[0,\frac{\pi}{4}\right]$, $\left[\frac{5\pi}{4},2\pi\right]$

(c) $f''(x)=e^x[-\sin x-\cos x]+e^x[\cos x-\sin x]$

$=-2e^x\sin x$

$f''(x)=0$ when $x=0,\pi,2\pi$

Point of inflection at $x=\pi$

5)

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$h'(x)$	$-$	0	$+$	und	$-$	0	$+$
x		$-\sqrt{2}$		0		$\sqrt{2}$	

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

(b) $h''(x) = 1 + \frac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

(c) $h'(4) = \frac{16-2}{4} = \frac{7}{2}$

$y + 3 = \frac{7}{2}(x - 4)$

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

6)

a) forward: (0,1) U (5,7)
backward: (1, 5) U (7, 9)
Speeding up: (1, 2) U (5, 6) U (7, 8)
Slowing down: (0,1) U (3, 5) U (6, 7)

b) positive (3, 6)
negative (0, 2) U (6, 8)
zero (2, 3) U (8,9)

c) on the interval (2, 3)

d) none