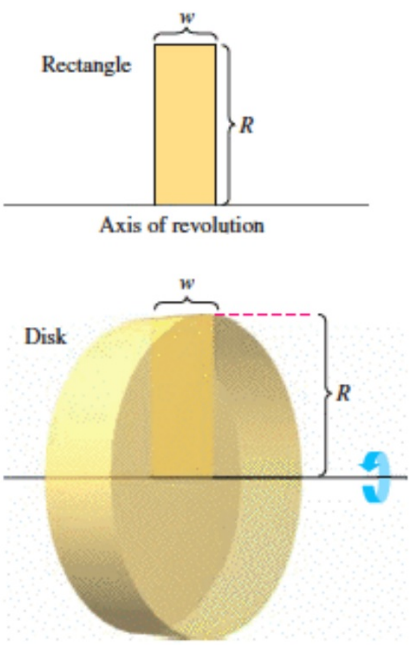
7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.



Volume of a disk: $\pi R^2 w$

$$V = \pi r^2 h$$

$$V = \pi \int_a^b r^2 \, dx$$

$$r^2 = (\text{top} - \text{bottom})^2$$
$$= (f(x) - \text{axis})^2$$

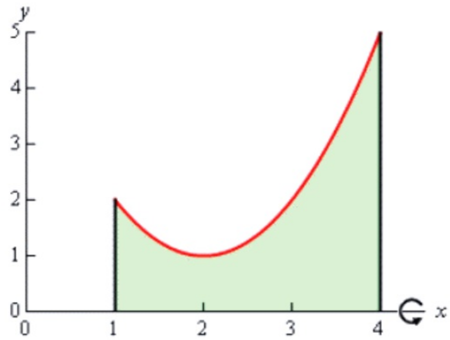
$$(x-4)^2$$

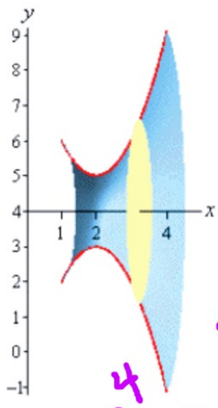
$$(4-x)^2$$

Volume by Revolution (Disk Method)

$$A = \pi\left(\left(\begin{array}{c}\text{outer}\\ \text{radius}\end{array}\right)^2 - \left(\begin{array}{c}\text{inner}\\ \text{radius}\end{array}\right)^2\right)$$

Example 1 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.





$y = x^2 - 4x + 5$
$x = 1$, $x = 4$,
rotate about the x-axis

Disk
- bounded region
- axis of rotation

$$R^2 = (x^2 - 4x + 5 - 0)^2$$

$$V = \pi \int_1^4 R^2\, dx$$

$$= \pi \int_1^4 (x^2 - 4x + 5)^2\, dx = 15.6\pi$$

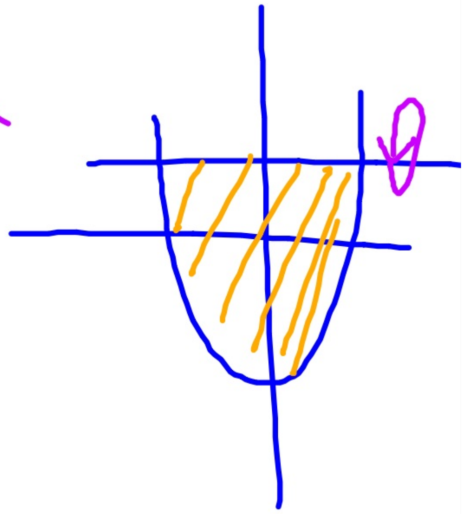
49.008 or 49.009

#2 Find the volume of the solid that is formed by $y = x^2 - 9$, $y = 4$; rotated about $y = 4$.

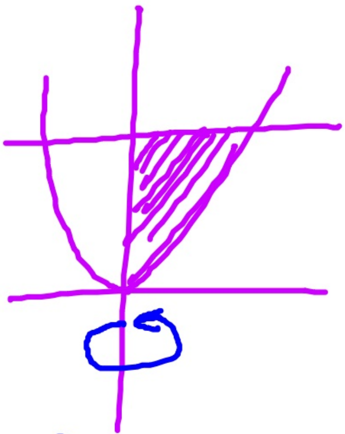
$$V = \pi \int_{-\sqrt{13}}^{\sqrt{13}} (x^2 - 13)^2 \, dx$$

2041.912

$$R^2 = (\text{func.} - \text{axis})^2$$
$$= (x^2 - 9 - 4)^2$$
$$= (x^2 - 13)^2$$
$$= (4 - (x^2 - 9))^2$$
$$= (13 - x^2)^2$$



#3 Region: $y = x^2$, $x = 0$, $y = 4$ (1st quad) rotated about y-axis



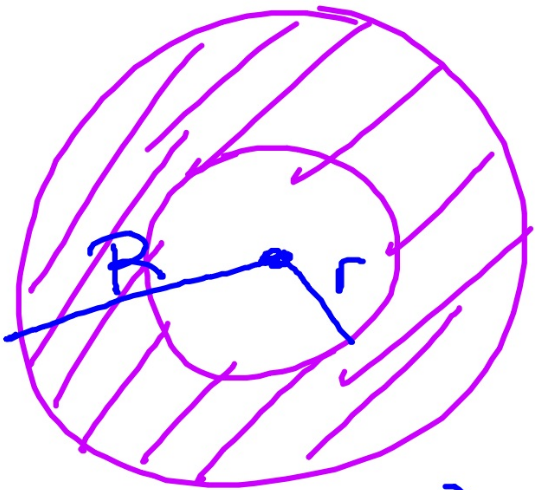
$$y = x^2$$
$$\pm\sqrt{y} = x$$

$$V = \pi \int_0^4 \sqrt{y}^{\,2}\, dy$$
$$= \pi \frac{y^2}{2} \Big]_0^4$$
$$= 8\pi$$

R^2 = Right − left

= funct. − axis

$= (\sqrt{y} - 0)^2$



$$\pi R^2 - \pi r^2$$

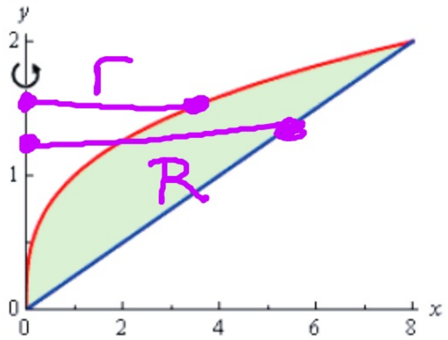
Volume by Revolution (Washer Method)

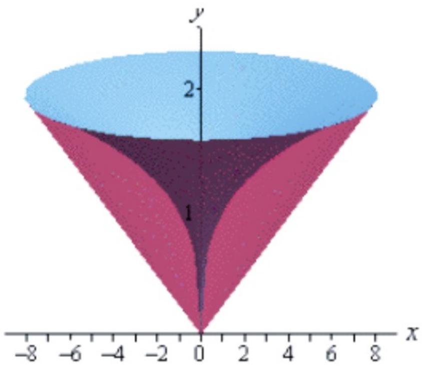
$$V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 \, dx$$

R^2 r^2

$$V = \pi \int_a^b \left[(outer - axis)^2 - (inner - axis)^2 \right] dx$$

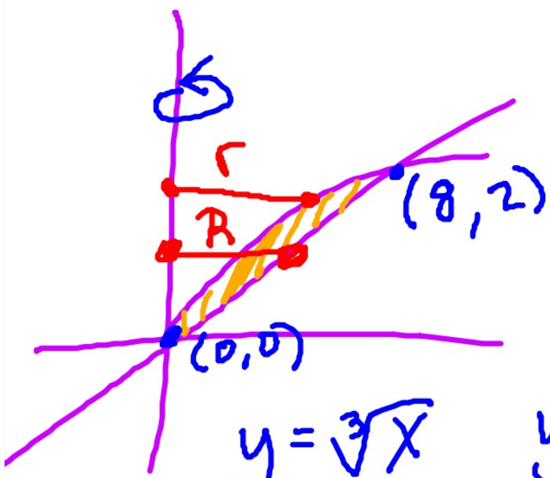
Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.





#4 $y=\sqrt[3]{x}$ $\quad y=\frac{x}{4}$ $\quad$ rotate : y-axis

1st Quad.



$$\pi\int_0^2\left(\left(4y-0\right)^2-\left(y^3-0\right)^2\right)dy$$

76.595

$y=\sqrt[3]{x}$ $\quad y=\frac{x}{4}$

$y^3=x$ $\quad x=4y$

$y^3-4y=0$ $\quad y^3=4y$

$y=0,\pm 2$

#5 *Find the volume of the solid obtained by rotating the region bounded by the curve* $y = x^2$ *and* $y = \sqrt{x}$ *about the line* $y = -1$.

$$\pi\int_0^1 \left(\left(\sqrt{x}--1\right)^2-\left(x^2--1\right)^2\right)dx$$

3.036
3.037

#6

Find the volume of the solid obtained by rotating the region bounded by $x = y^2 - 2$ and $y = x$ about the line x=4

parabola

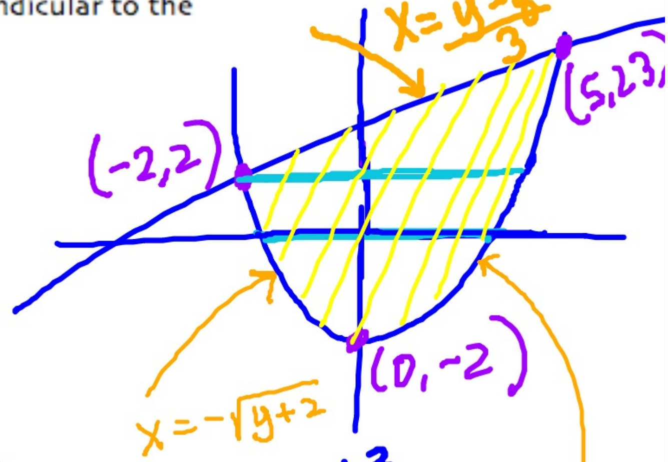
$$\pi \int_{-1}^{2} \left(\left(y^2 - 2 - 4 \right)^2 - \left(y - 4 \right)^2 \right) dy$$

124.407

x	y
-1	1
-2	0
-1	-1

3. (Calculator) The base of a solid is the region bounded by $y = x^2 - 2$ and $y = 3x + 8$. Find the volume of each solid whose cross sections perpendicular to the

a. x-axis are semicircles
b. y-axis are squares
c. y-axis are equilateral triangles
d. x-axis are rectangles with height of 4



$$\int_a^b s^2\,dy$$

$$\int_{-2}^{2}\left(\sqrt{y+2} - -\sqrt{y+2}\right)^2 dy + \int_{2}^{23}\left(\sqrt{y+2} - \frac{y-8}{3}\right)^2 dy$$

$$\int_{-2}^{2}\left(2\sqrt{y+2}\right)^2 + \int_{2}^{23}\left(\sqrt{y+2} - \frac{y-8}{3}\right)^2 dy$$

AP Practice Question

ıe shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and

line $x = 1$, as shown in the figure above.

ıe area of R.

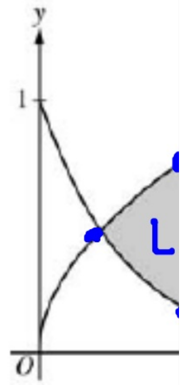
ıe volume of the solid generated when R is revolved about the horizontal

$= 1$.

gion R is the base of a solid. For this solid, each cross section

dicular to the x-axis is a rectangle whose height is 5 times the length of its

region R. Find the volume of this solid.



c.) $\int_a^b \text{Area}\,dx = \int_{.239}^{1} 5(\sqrt{x} - e^{-3x})^2\,dx = 1.554$

$L = \sqrt{x} - e^{-3x}$

$H = 5(\sqrt{x} - e^{-3x})$

$LH = 5(\sqrt{x} - e^{-3x})^2$

↑ Area

Answers to AP Question

Point of intersection

$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$

$= 0.442$ or 0.443

(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2 \right) dx$

$= 0.453\pi$ or 1.423 or 1.424

(c) Length $= \sqrt{x} - e^{-3x}$

Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

1: Correct limits in an integral in (a), (b), or (c)

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

$3: \begin{cases} 2: \text{integrand} \\ \quad <-1> \text{ reversal} \\ \quad <-1> \text{ error with constant} \\ \quad <-1> \text{ omits 1 in one radius} \\ \quad <-2> \text{ other errors} \\ 1: \text{answer} \end{cases}$

$3: \begin{cases} 2: \text{integrand} \\ \quad <-1> \text{ incorrect but has } \sqrt{x} - e^{-3x} \text{ as a factor} \\ 1: \text{answer} \end{cases}$