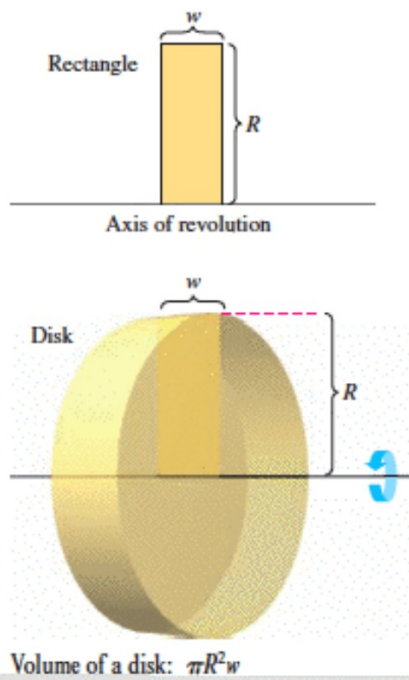


7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.



$$V = \pi r^2 h$$

$$V = \pi \int_a^b r^2 dx$$

$$r^2 = (\text{top} - \text{bottom})^2$$
$$= (f(x) - \text{axis})^2$$

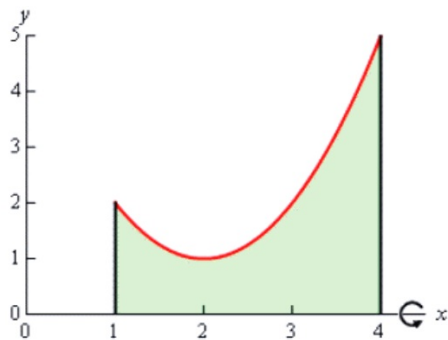
$$(x-4)^2$$

$$(4-x)^2$$

Volume by Revolution (Disk Method)

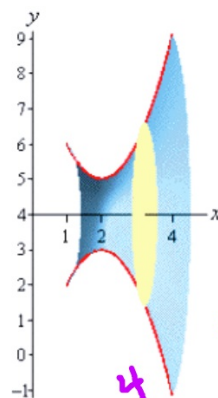
$$A = \pi \left(\left(\begin{matrix} \text{outer} \\ \text{radius} \end{matrix} \right)^2 - \left(\begin{matrix} \text{inner} \\ \text{radius} \end{matrix} \right)^2 \right)$$

Example 1 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x -axis about the x -axis.



Disk

- bounded region
- axis of rotation



$y = x^2 - 4x + 5$
 $x = 1, x = 4$,
 rotate
 about the
 x-axis

$$R = (x^2 - 4x + 5 - 0)^2$$

$$V = \pi \int_1^4 R^2 dx$$

$$= \pi \int_1^4 (x^2 - 4x + 5)^2 dx = 15.6\pi$$

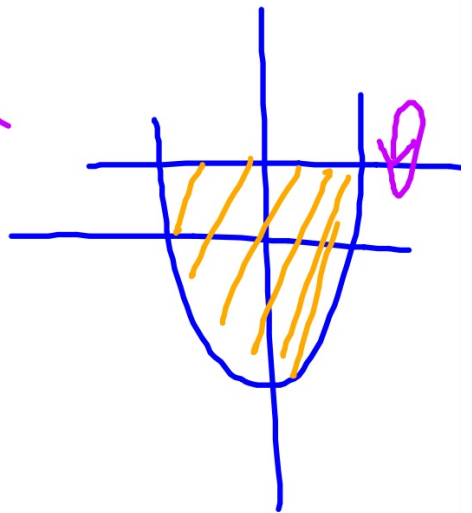
49.008 or
49.009

#2 Find the volume of the solid that is formed by $y = x^2 - 9$, $y = 4$; rotated about $y = 4$.

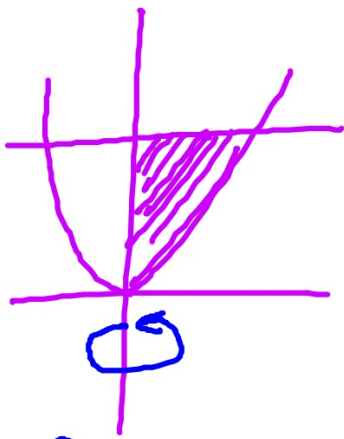
$$V = \pi \int_{-\sqrt{13}}^{\sqrt{13}} (x^2 - 13)^2 dx$$

$$\begin{aligned} R^2 &= (\text{func.} - \text{axis})^2 \\ &= (x^2 - 9 - 4)^2 \\ &= (x^2 - 13)^2 \\ &= (4 - (x^2 - 9))^2 \\ &= (13 - x^2)^2 \end{aligned}$$

2041.912



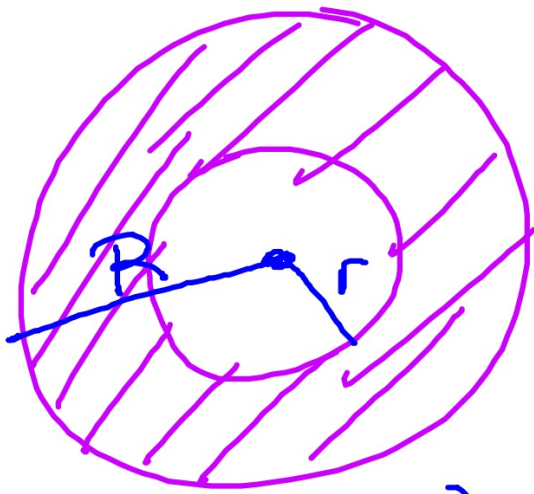
#3 Region: $y = x^2$, $x = 0$, $y = 4$ (1st quad)
rotated about y-axis



$$y = x^2$$
$$\pm\sqrt{y} = x$$

$$V = \pi \int_0^4 \sqrt{y}^2 dy$$
$$= \pi \left. \frac{y^2}{2} \right|_0^4$$
$$= 8\pi$$

$$R^2 = \text{Right} - \text{left}$$
$$= \text{funct.} - \text{axis}$$
$$= (\sqrt{y} - 0)^2$$



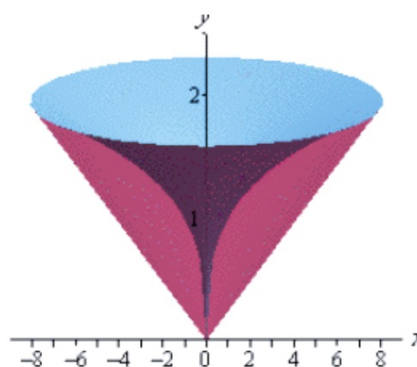
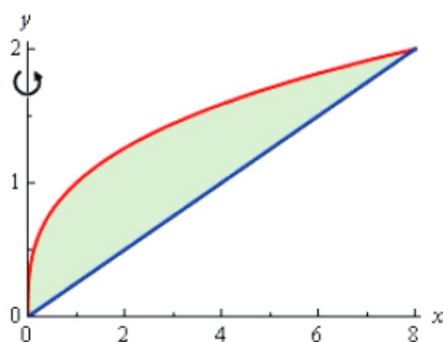
$$\pi R^2 - \pi r^2$$

Volume by Revolution (Washer Method)

$$V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 dx$$

$$V = \pi \int_a^b [(outer - axis)^2 - (inner - axis)^2] dx$$

Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y -axis.

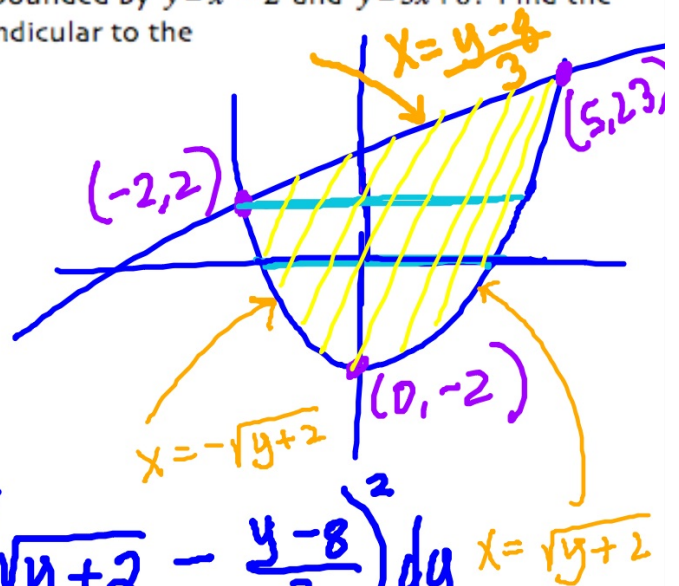


#4 Find the volume of the solid obtained by rotating the region bounded by the curve $y = x^2$ and $y = \sqrt{x}$ about the line $y = -1$.

#5 Find the volume of the solid obtained by rotating the region bounded by $x = y^2 - 2$ and $y = x$ about the line $x=4$

3. (Calculator) The base of a solid is the region bounded by $y = x^2 - 2$ and $y = 3x + 8$. Find the volume of each solid whose cross sections perpendicular to the

- a. x -axis are semicircles
- b. y -axis are squares**
- c. y -axis are equilateral triangles
- d. x -axis are rectangles with height of 4



$$\int_a^b s^2 dy$$

$$\int_{-2}^2 (\sqrt{y+2} - (-\sqrt{y+2}))^2 dy + \int_2^{23} (\sqrt{y+2} - \frac{y-8}{3})^2 dy$$

$$\int_{-2}^2 (2\sqrt{y+2})^2 dy + \int_2^{23} (\sqrt{y+2} - \frac{y-8}{3})^2 dy$$

Let region R be enclosed by
 $f(x) = 1/x^2$; $x = 1$, $x = 4$, x-axis

a) Find the area of region R.

b) Find the number a such that line $x = a$ bisects region R.